Exotic atoms with strangeness

E. Friedman

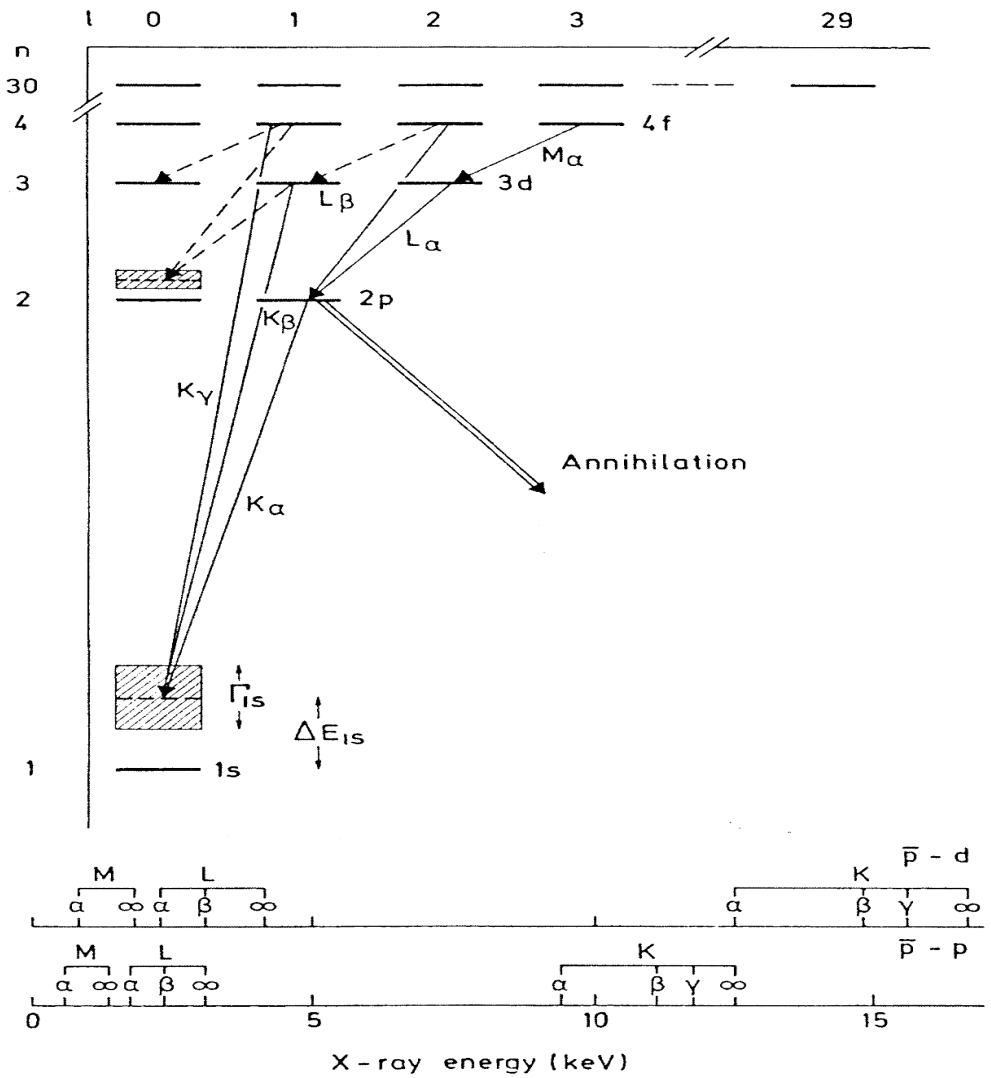
Racah Institute of Physics, Hebrew University, Jerusalem

22^{nd} Indian-Summer School and SPHERE school,
Rez/Prague September 2010

OUTLINE

- Overview of exotic atoms.
- Phenomenology and global fits.
- K^- atoms: many details, surprizes and puzzles.
- Σ^- atoms: little data but firm conclusions.
- Towards Ξ^- atoms.
- Summary

The classical method



Different scenarios for different exotic atoms

particle	real potl.	imaginary potl.	comments
π^-	repulsive in bulk attractive on surface	moderate	excellent data well understood
$\bar{p}$	model dependent	very absorptive	excellent data understood
K^-	attractive deep or shallow?	moderate	good data open problems
Σ^-	repulsive in bulk attractive outside	moderate	limited data poorly understood

Ξ^-: initial estimates Batty *et.al*, 1999; J-PARC proposal

Phenomenological analyses of data:

- handle large sets of data
- Could identify characteristic quantities
- serve as intermediaries between 'genuine' theories and experiment (e.g. in reproducing the characteristic quantities)

Tools of the trade: variants of an optical potential.

When analyzing several nuclear species together one must have some model for the nuclear geometry, e.g. make the potential a functional of the nuclear density.

The simplest class of optical potentials $V_{\rm opt}$ is the generic $t\rho(r)$ potential: (isoscalar)

$$2\mu V_{\rm opt}(r) = -4\pi(1 + \frac{A-1}{A}\frac{\mu}{M})b_0[\rho_n(r) + \rho_p(r)]$$

ρ_n and ρ_p are neutron and proton densities normalized to N and Z, respectively, M is the mass of the nucleon.

Results of global fits apply to average behaviour.

The strong interaction with the nucleus is confined to a small region compared to the size of the (exotic!) atom, e.g. ≈ 4 fm out of ≈ 40 fm, hence perturbation approach is inapplicable.

Must solve the wave equation with and without the strong interaction and take the difference between the *complex* energies.

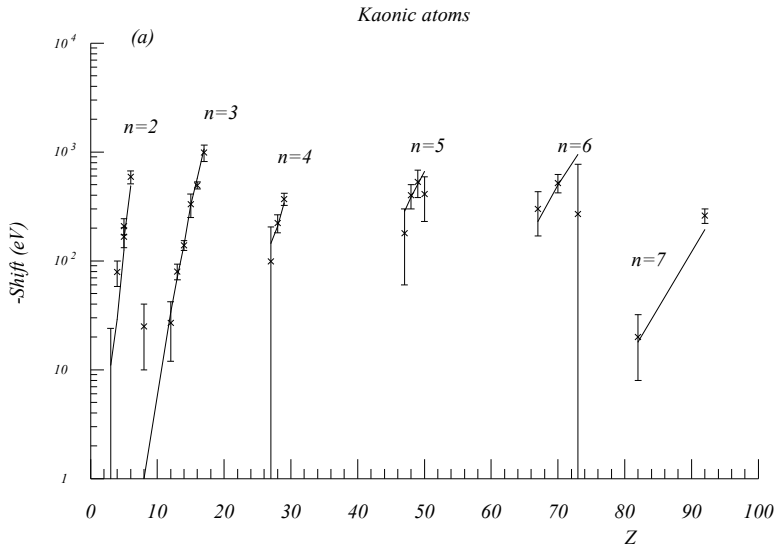
Numerical accuracy required is typically 10^{-5}.

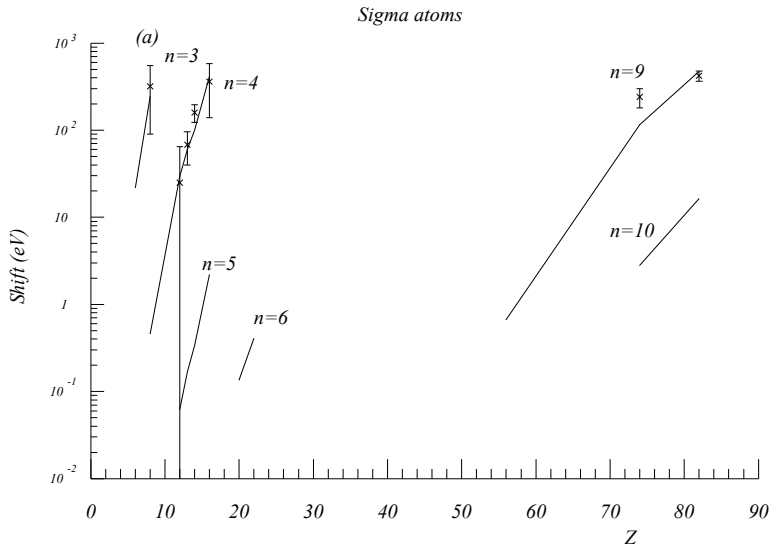
For Bosons use the Klein-Gordon equation

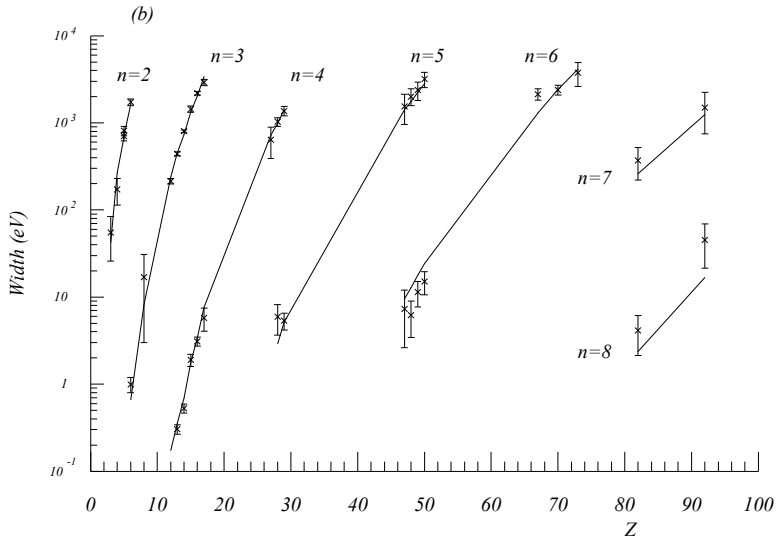
$$\left[\nabla^2 - 2\mu(B + V_{\rm opt} + V_c) + (V_c + B)^2\right]\psi = 0 \quad (\hbar = c = 1)$$

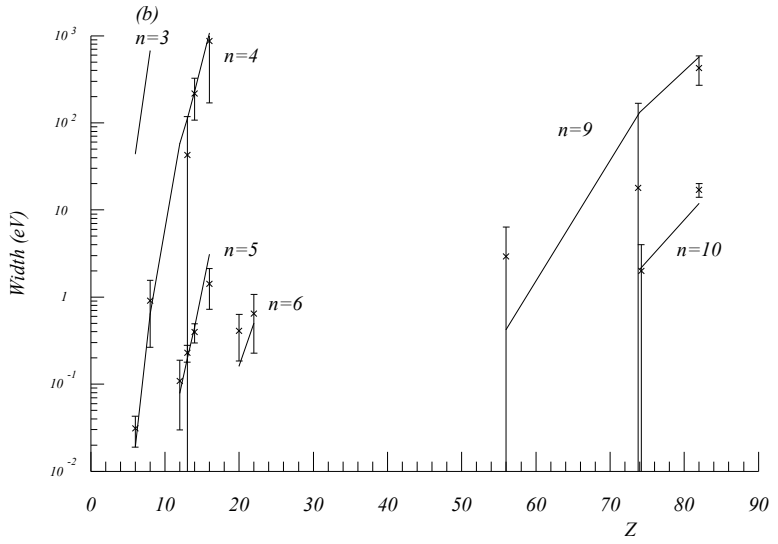
where μ is the hadron-nucleus reduced mass, B is the complex binding energy and V_c is the finite-size Coulomb interaction of the hadron with the nucleus, including vacuum-polarization terms, added according to the minimal substitution principle $E \to E - V_c$. A term $2V_cV_{\rm opt}$ and a term $2BV_{\rm opt}$ were neglected with respect to $2\mu V_{\rm opt}$.

Need to use the Dirac equation for Fermions, but for a given l value the j-averaged energies are equal to the corresponding Klein-Gordon energies. Numerically this is the case to better than 10% of the smallest experimental errors for antiprotons.

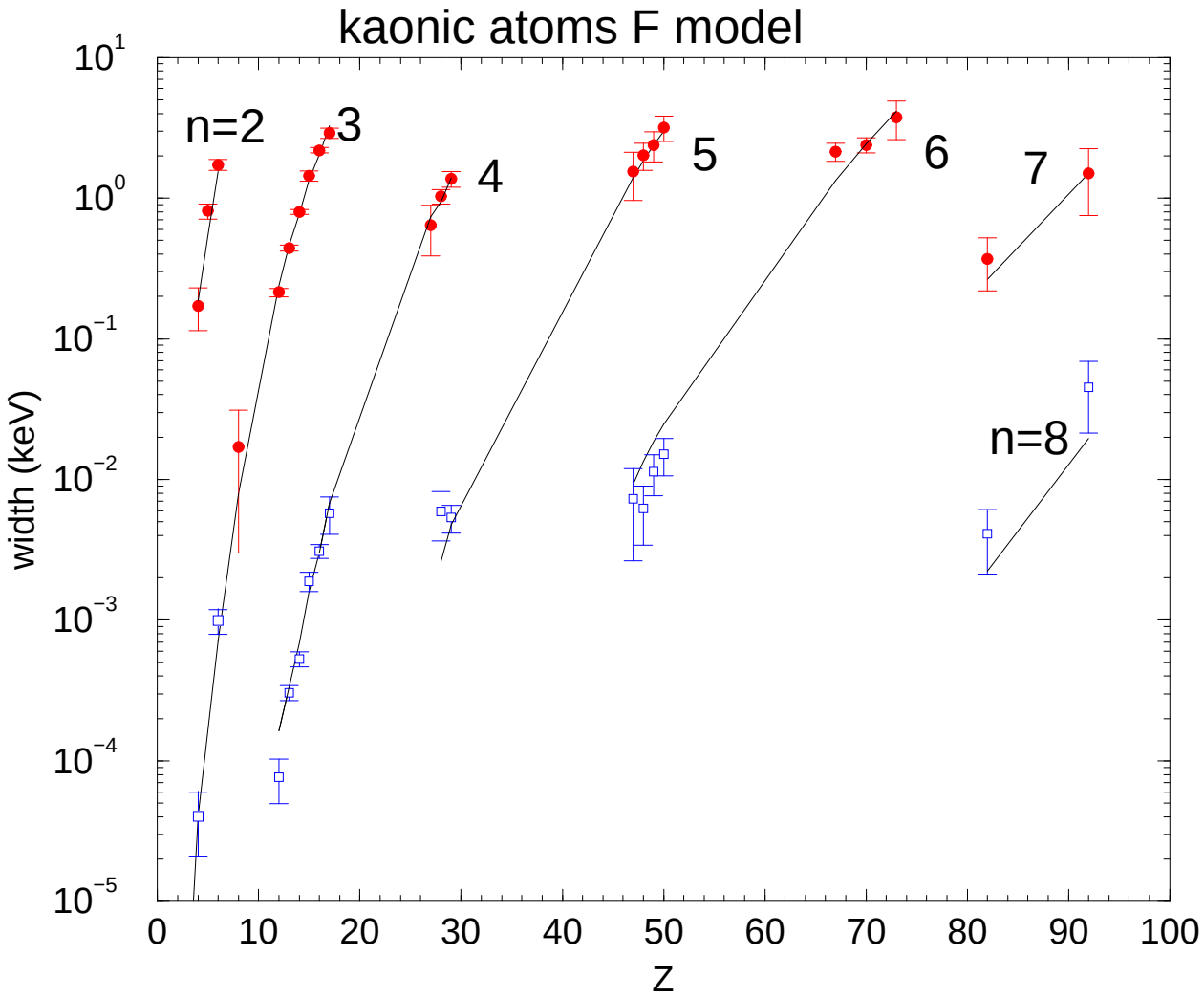






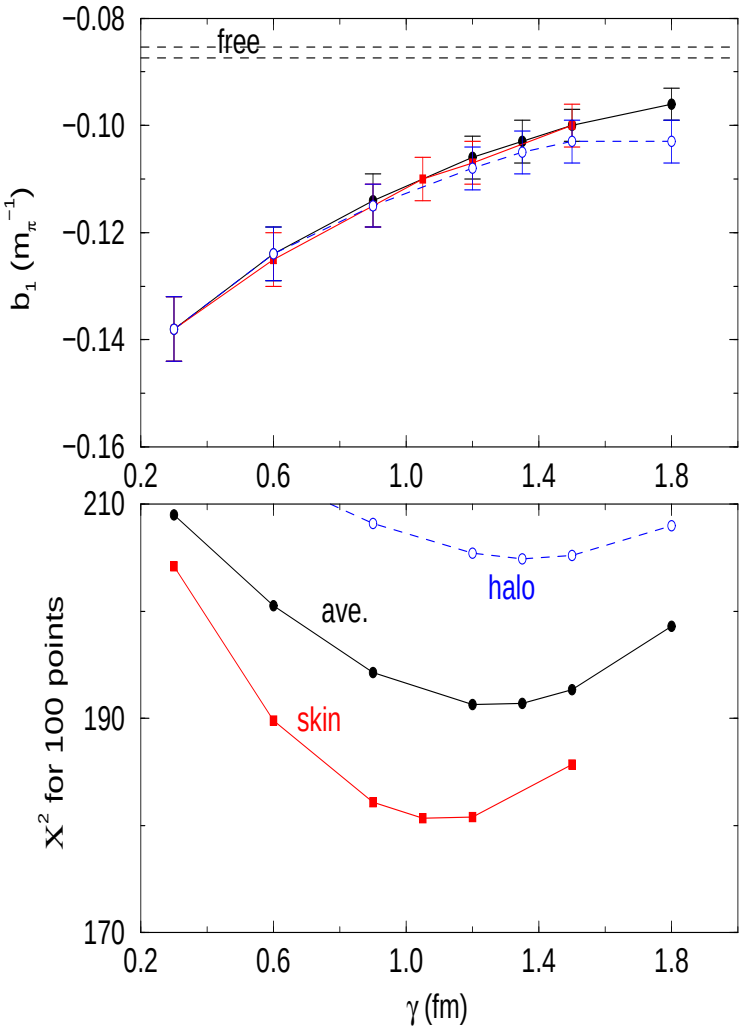


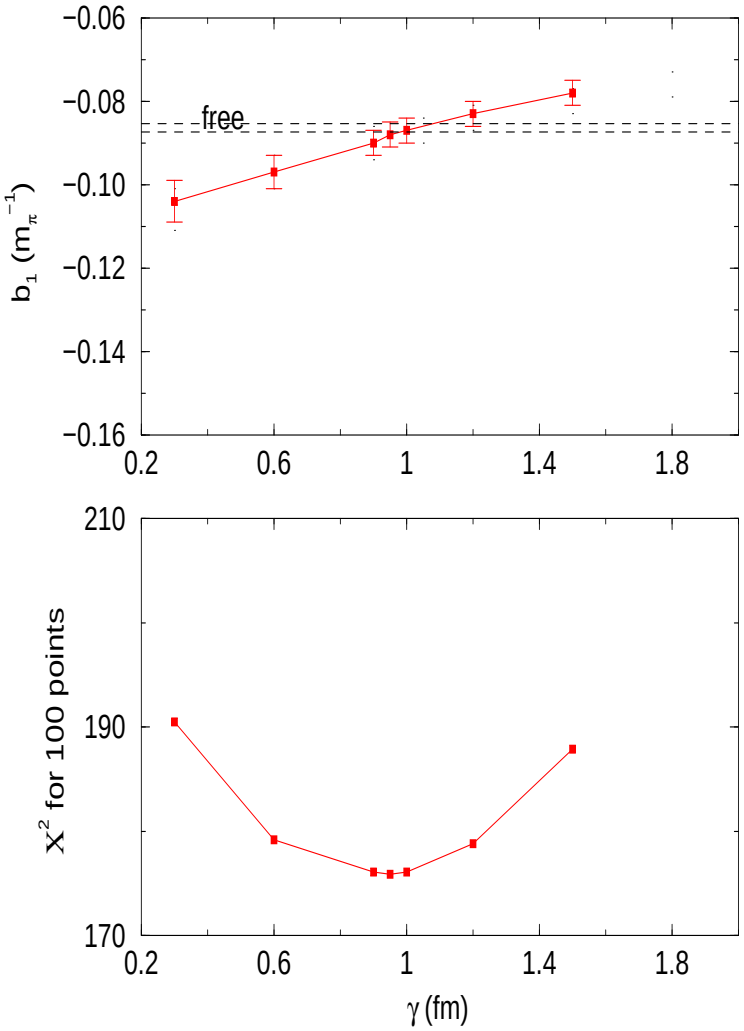
Data and fits for kaonic atoms (left) and Σ^- atoms (right).



Quality fits over five orders of magnitude.

The lesson from pionic atoms: partial restoration of chiral symmetry?





The best-fit b_1 value disagrees with the free πN value.

From the Tomozawa-Weinberg expression

$$b_1(\rho) = -\frac{\mu_{\pi N}}{8\pi f_\pi^2(\rho)}$$

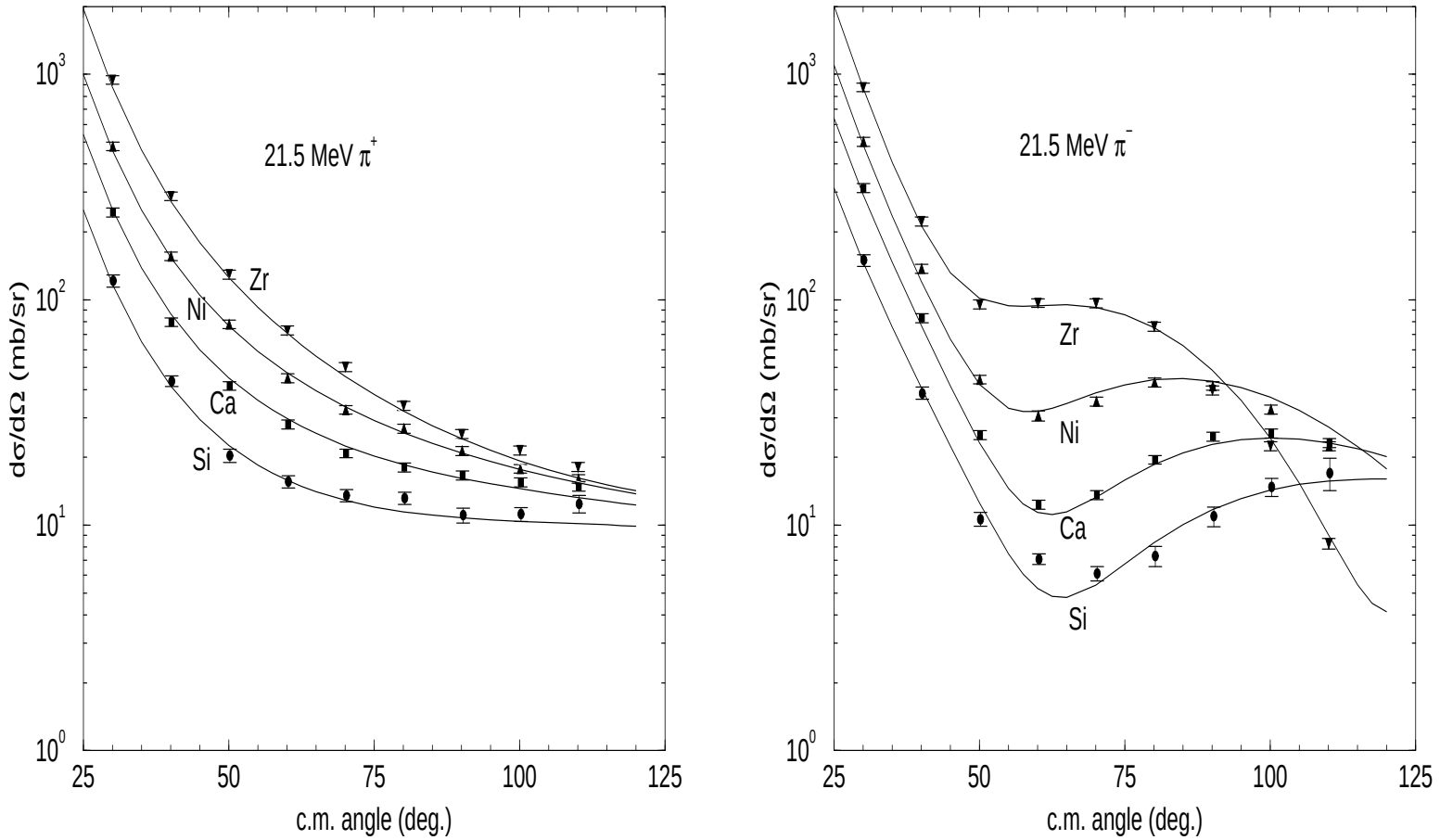
and with Weise's in-medium extension

$$\frac{f_\pi^2(\rho)}{f_\pi^2} = \frac{< \bar{q}q >_\rho}{< \bar{q}q >_0} \simeq 1 - \frac{\sigma\rho}{m_\pi^2 f_\pi^2}$$

one gets a density dependendent $b_1(\rho)$

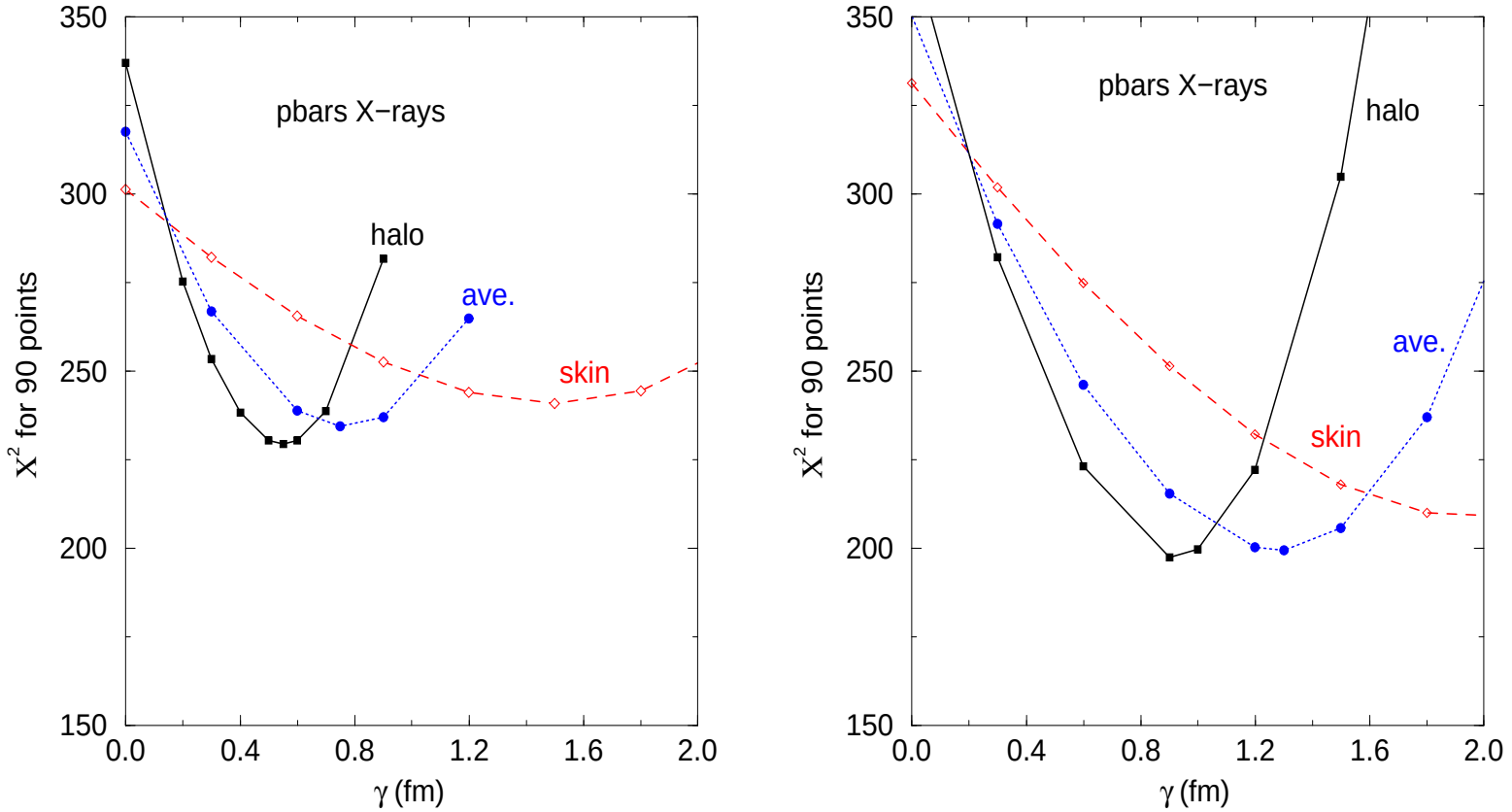
$$b_1(\rho) = \frac{b_1(0)}{1 - \frac{\sigma\rho}{m_\pi^2 f_\pi^2}} \,,$$

and then $b_1(0)$ agrees with the free value!



Low energy $\pi^{\pm}$ nucleus elastic scattering angular distributions reproduced with the $b_1(\rho)$ ansatz.

What is expected?



Zero range (left) and finite range (right) fits to strong interaction shifts and widths in antiprotonic atoms. For the latter the best fit is obtained for a rms radius of the $\bar{p}$N interaction of 1.1±0.1 fm.

Global fits to kaonic atoms data (65 points)

model	χ^2	$-\mathrm{Re}V(0)$ (MeV)	$-\mathrm{Im}V(0)$ (MeV)
$t\rho$	130	81($\pm$10%)	122($\pm$5%)
$t(\rho)\rho$	84	180($\pm$3.5%)	82($\pm$8%)
chiral *	266	33	45
chiral **	120	42	62

*Ramos & Oset, NPA **671** (2000) 481

** I=1 adjusted by +50% and +63% for Re and Im, respectively

The DD form is based on modifying the effective scattering length b_0

$$b_0 \rightarrow b_0 \; + \; B_0 \, \{\frac{\rho(r)}{\rho_0}\}^{\alpha} \; , \quad \alpha > 0 \; ,$$

where $\rho_0 = 0.16$ fm^{-3} is a central nuclear density. It is possible then to respect the 'low density limit' by keeping b_0 fixed, $b_0 = b_0^{\rm free}$, varying the parameters B_0 and α.

The F form is based on modifying b_0 as follows:

$$b_0 \;\rightarrow\; B_0\, F(r) \;+\; b_0\,[1 \;-\; F(r)] \quad .$$

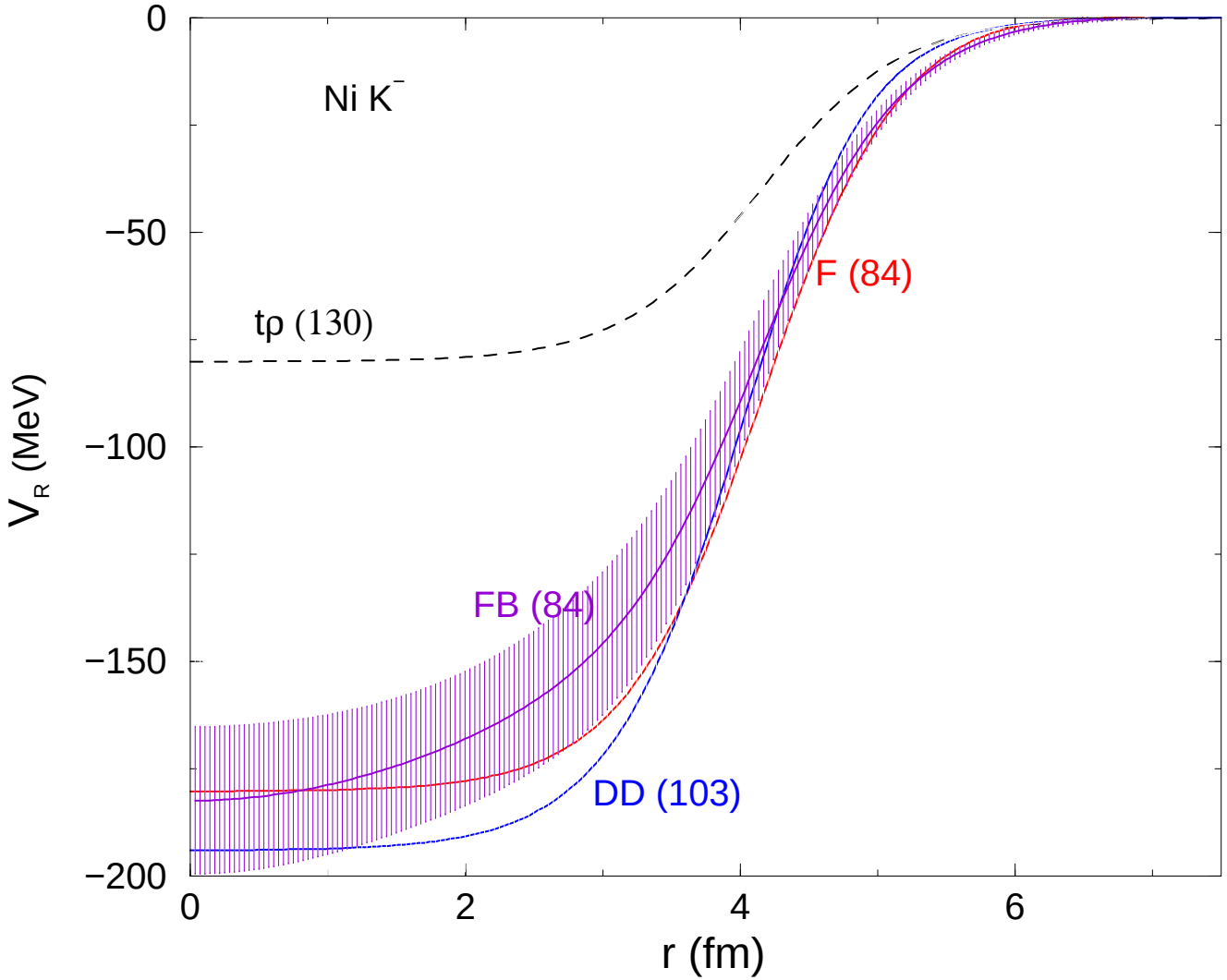
The density-like function $F(r)$ is defined as

$$F(r) \;=\; \frac{1}{e^x + 1}\;, \qquad x \;=\; \frac{r - R_x}{a_x}\;.$$

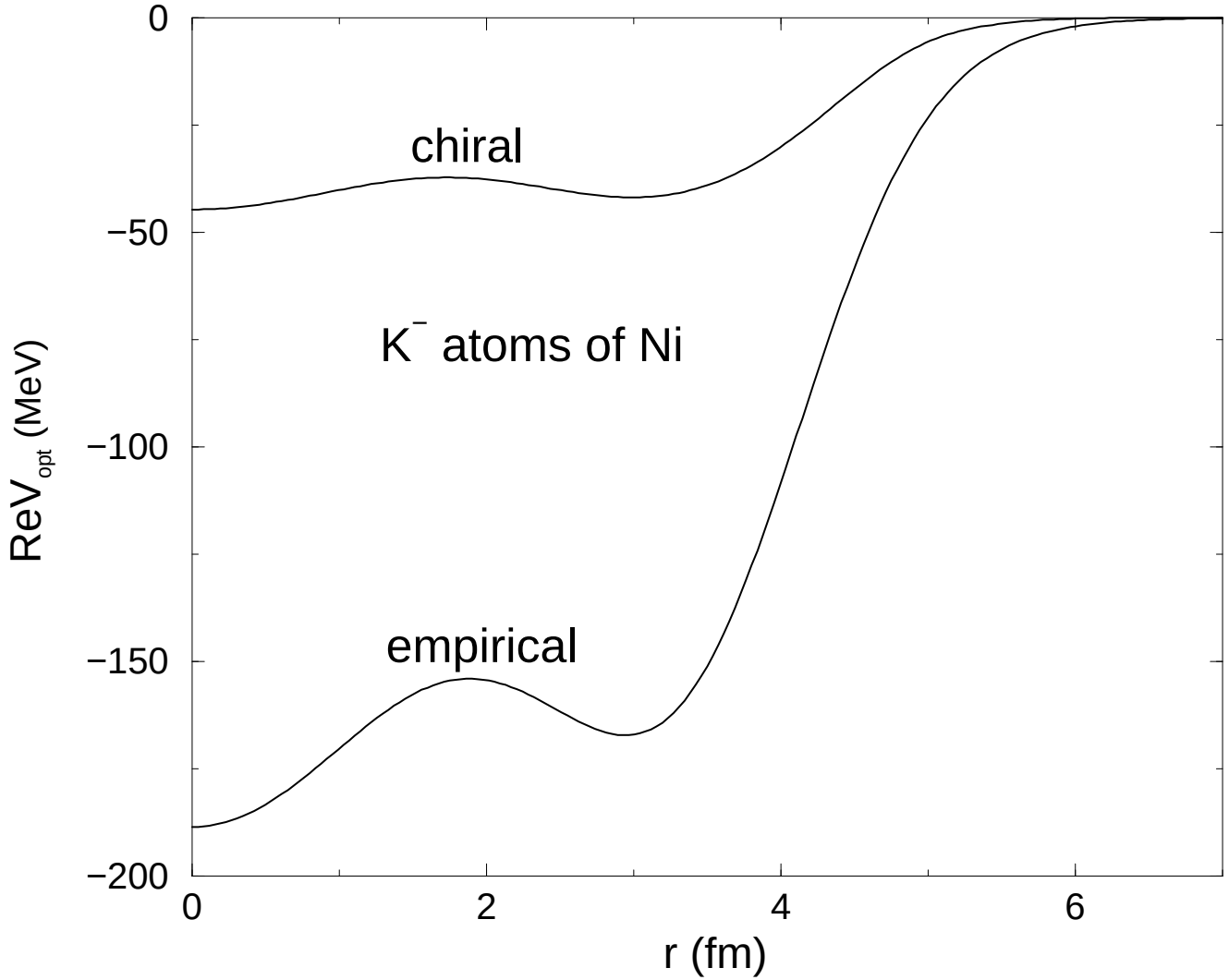
$F(r) \rightarrow 1$ for $r << R_x$ defining an internal region and similarly $[1 - F(r)] \rightarrow 1$ for $r >> R_x$ defining an external region. *If* R_x is close to the nuclear surface then the two regions correspond to the high-density and low-density regions of nuclei, respectively.

In global fits across the periodic table, R_x is chosen as $R_x = R_{x0}A^{1/3} + \delta_x$ and the parameters B_0, R_{x0} and δ_x are varied in the least-squares fit, while gridding on values of a_x around 0.5 fm.

From best fits $R_{x0} = 1.30 \pm 0.05$ fm, $\delta_x = 0.8 \pm 0.3$ fm thus confirming the separation into 'internal' and 'external' regions.



In brackets values of χ^2 for 65 data points.

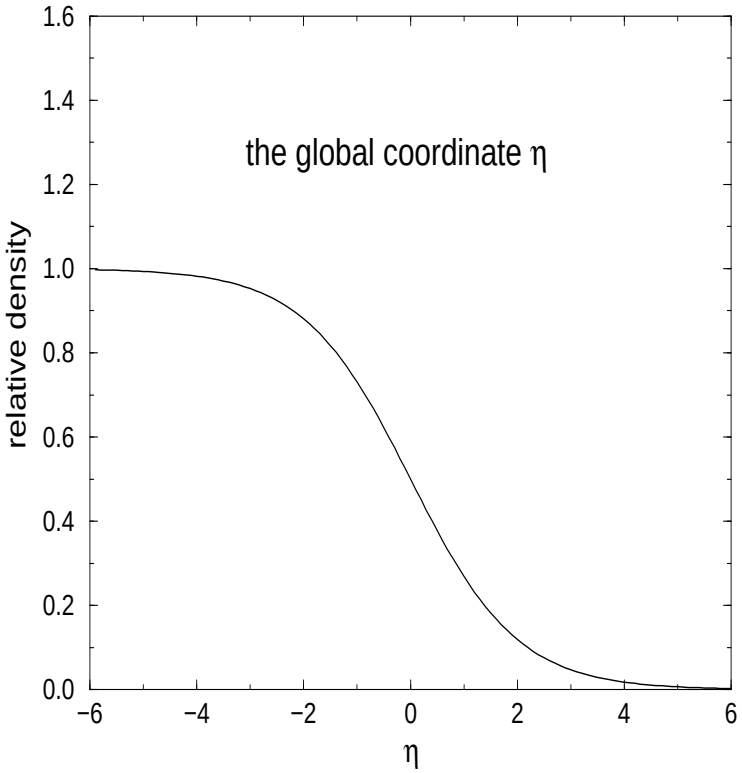

0
−50
−100
−150
−200
ReV$_{opt}$ (MeV)
chiral
K$^-$ atoms of Ni
empirical
0
2
4
6
r (fm)

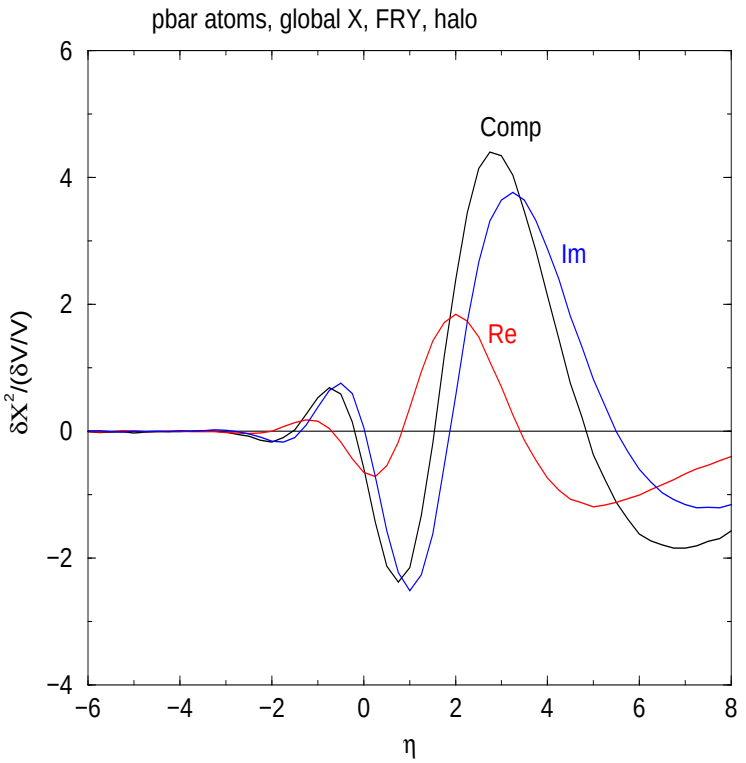
Extensions with finite-range (FR) folding?

- DD and F real potentials have sharper surface compared to ρ (smaller rms radii than that of ρ).
- No improvement in fits when FR applied to a $t\rho$ potential.
- Applying FR to DD or F potentials leads to zero range.

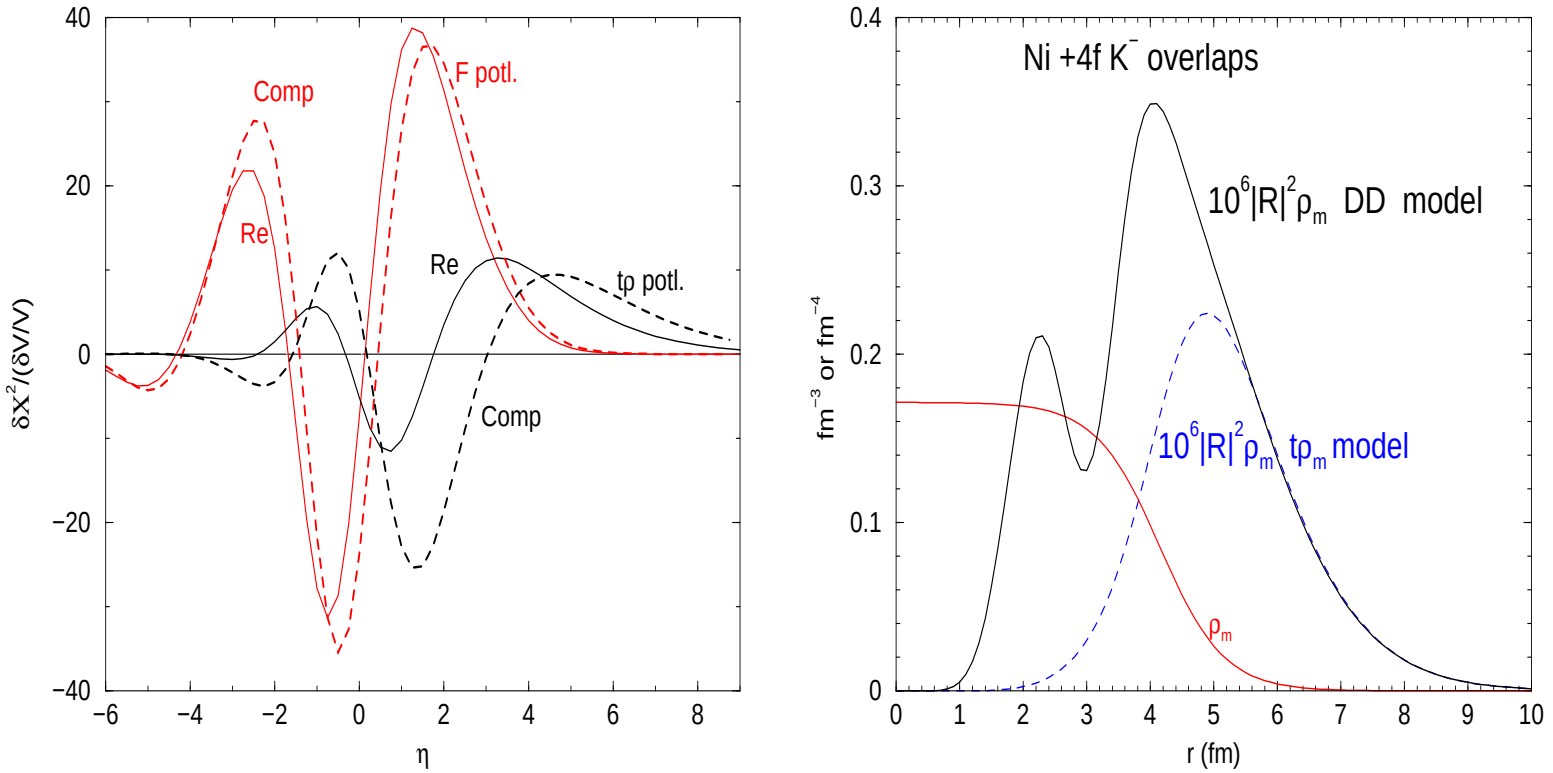
Do *exotic* atoms experiments probe the interior?

The functional derivative method





Left: Define η by $r = R_c + \eta\, a_c$. The value of χ^2 becomes a functional of a global optical potential $V(\eta)$. Right: Functional derivatives of χ^2 for $\bar{p}$ atoms (Phys. Rev. C **75** (2007) 022202(R)).

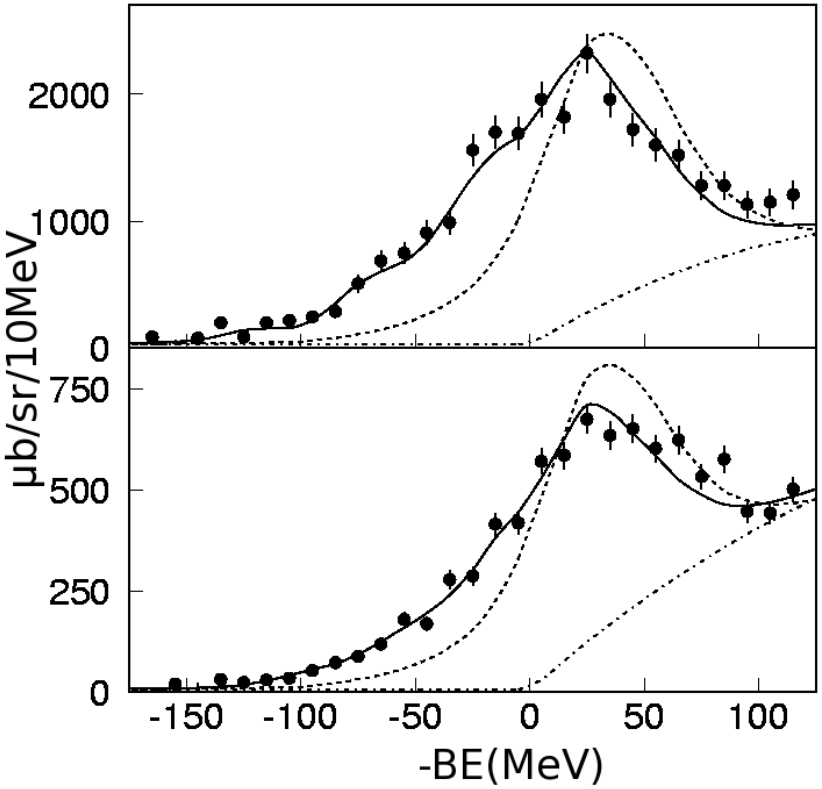


Left: Functional derivatives for kaonic atoms χ^2.

Right: Overlaps of K^- atomic density with the nuclear density.

R_B =31.5 fm.

Supporting evidence for a deep potential, Kishimoto *et.al*, (2007)



KEK-PS E548 missing mass (K^-, n) (upper) & (K^-, p) (lower) spectra on ^{12}C at $p_{K^-} = 1$ GeV/c.

Is there an experimental problem?

Focusing on targets with large χ^2 for the shallow potential:

- conflicting χ_Γ and χ_Y, i.e. no systematics
- when removed from data base, still the same two solutions (deep and shallow)

The two solutions are inherent property of the data.

Comparing full and 'less' data sets

N	χ^2	Reb(fm)	Imb(fm)	χ^2	ReB(fm)	ImB(fm)
65	130	0.62±0.05	0.93±0.04	84	1.44±0.03	0.59±0.03
56	78	0.57±0.05	0.97±0.04	66	1.44±0.04	0.60±0.04
		shallow			deep	

Removing data for C, Mg and Si (three different experiments!) the two solutions are still there.

Repeating some of the K^- atom experiments?

Typical quantities for the reduced set of kaonic atoms

target	C	Si	Ni	Sn	Pb
ref	(a)	(b)	(b),(c)	(b)	(d)
(n,l)	2p	3d	4f	5g	7i
$-\epsilon$ (keV)	0.50 ± 0.08	0.130 ± 0.015	0.223 ± 0.042	0.41 ± 0.18	0.020 ± 0.012
Γ (keV)	1.73 ± 0.15	0.800 ± 0.033	1.03 ± 0.12	3.18 ± 0.64	0.37 ± 0.15
yield	0.070 ± 0.013	0.49 ± 0.03	0.30 ± 0.08	0.39 ± 0.07	0.70 ± 0.08
Γ_u (eV)	0.99 ± 66	0.53 ± 0.06	5.9 ± 2.3	15.1 ± 4.4	4.1 ± 2.0
EM n+1→n					
energy (keV)	63.3	123.7	231.6	403.9	426.2

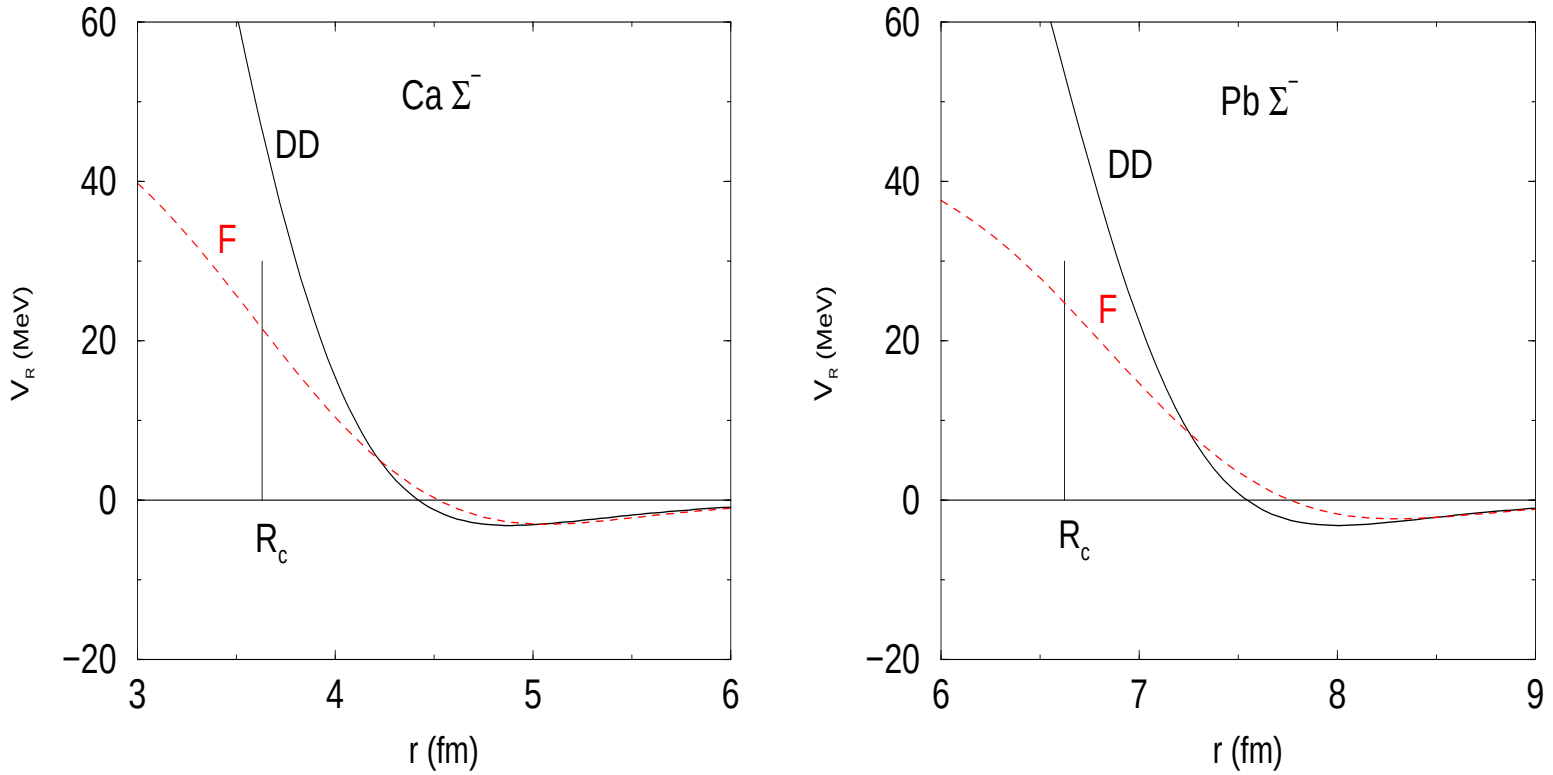
(a) PLB **38** 181 (1972)
(b) NPA **329** 407 (1979)
(c) NPA **231** 477 (1974)
(d) NPA **254** 381 (1975)

Full and reduced K^- atom data set fits. The reduced set consists of $2p, 3d, 4f, 5g, 7i$ shifts, widths and yields in C, Si, Ni, Sn and Pb targets, respectively

	shallow potential			deep potential		
N	Re $b(\rho_0)$	Im $b(\rho_0)$	χ^2	Re $b(\rho_0)$	Im $b(\rho_0)$	χ^2
65	0.62±0.05	0.93±0.04	130	1.44±0.03	0.59±0.03	84
15	0.78±0.13	0.92±0.11	44	1.47±0.05	0.55±0.06	26

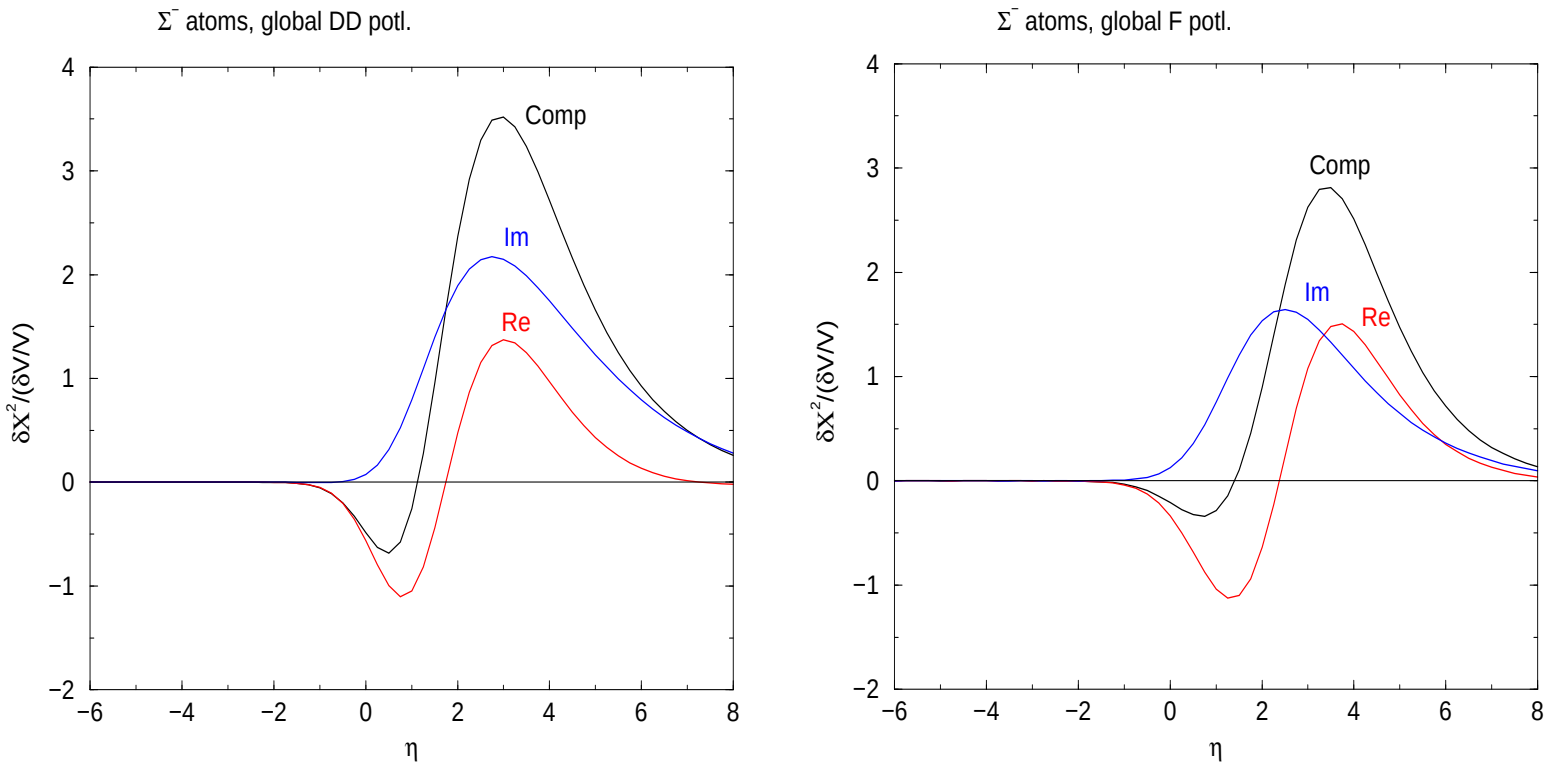
Σ^- atoms

- 24 data points, limited accuracy, (K^- background).
- $t\rho$ fits, $\chi^2 \approx 48$, attractive and absorptive potential.
- DD fits (1994), $\chi^2 \approx 20$, weak attraction outside, repulsion inside.
- F-model fits (2007), $\chi^2 \approx 20$, weak attraction outside, repulsion inside (model-dependent).
- $(K^-, \pi^\pm)$ spectra provide credible evidence that Σ hyperons generally do not bind in nuclei.



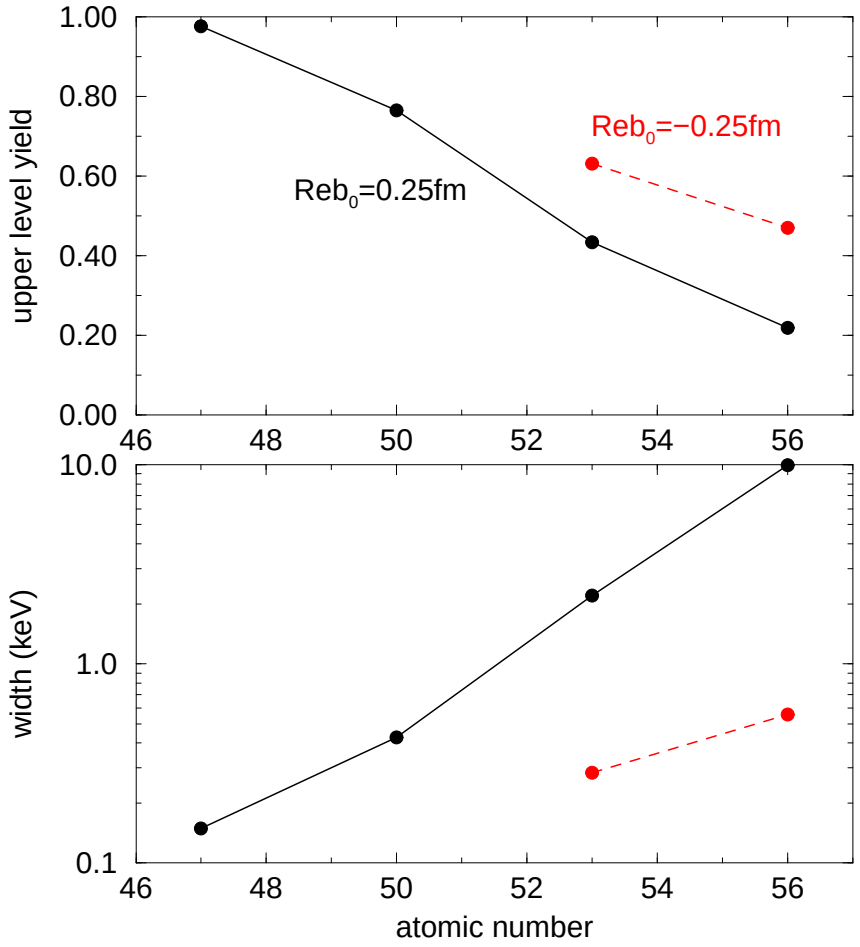
Real potentials from global fits to Σ^- atoms.

R_c is the half-density radius of the charge distribution.



Functional derivatives of χ^2 for Σ^- atoms.

Experiments with Ξ^- atoms? (Batty, Friedman and Gal, PRC **59** (1999) 295)



Calculated strong interaction widths and upper level relative yields for the $7i$ level.

In choosing criteria for the suitability of a transition as a source of information on the Ξ nucleus interaction, we are guided by experience with other hadronic atoms and select X-ray transitions $(n+1, l+1) \rightarrow (n, l)$ between circular atomic states $(n = l + 1)$:

- transition energies greater than 100 keV
- 'lower' level strong interaction shift at least 0.5 keV
- 'lower' level width less than about 10 keV
- 'upper' level relative yield at least 10%

Predictions for likely targets for a Ξ^- atoms experiment.
Calculations are based on a $t\rho$ potential with $b_0 = 0.25 + i0.04$ fm.
E_x is the transition energy, Y is the upper level relative yield.

target	F	Cl	Sn	I	Pb
transition	$4f \to 3d$	$5g \to 4f$	$8j \to 7i$	$8j \to 7i$	$10l \to 9k$
E_x (keV)	131.29	223.55	420.25	474.71	558.47
Y	0.31	0.37	0.76	0.43	0.58
shift (keV)	1.56	1.84	0.67	2.79	1.73
width (keV)	0.99	1.14	0.43	2.21	1.26

J-PARC proposal

Calculated binding energies, shifts and widths for Ξ^- atoms of ^{56}Fe

state	B(point) (keV)	B(FS+VP) (keV)	typical shift (keV)	typical width (keV)
$5g_{7/2}$	928.446	933.347	0.7-1.5	0.25-0.85
$5g_{9/2}$	928.112	933.007		
$6h_{9/2}$	644.580	647.356	0.006-0.011	0.002-0.004
$6h_{11/2}$	644.451	647.225		
$7i_{11/2}$	473.501	475.160	0.	0.
$7i_{13/2}$	473.443	475.101		

Typical radiation yields: $6h \rightarrow 5g \;=\; 50\%, \quad 7i \rightarrow 6h \;=\; 100\%$

J-PARC proposal

Calculated binding energies, shifts and widths for Ξ^- atoms of ^{58}Ni

state	B(point) (keV)	B(FS+VP) (keV)	typical shift (keV)	typical width (keV)
$5g_{7/2}$	1077.822	1083.715	1.2-2.8	0.45-1.85
$5g_{9/2}$	1077.372	1083.259		
$6h_{9/2}$	748.253	751.626	0.010-0.020	0.007-0.012
$6h_{11/2}$	748.079	751.450		
$7i_{11/2}$	549.645	551.675	0.	0.
$7i_{13/2}$	549.567	551.596		

Typical radiation yields: $6h \rightarrow 5g \;=\; 35\%, \quad 7i \rightarrow 6h \;=\; 100\%$

Summary: Exotic atoms with strangeness

- Unique source of information on *average* behaviour at threshold.
- Kaonic atoms: very significant medium-modification of the K^--nucleon interaction. A puzzle?
- Σ^- atoms: very shallow attraction far outside nuclei. Well-established repulsion from the surface inwards. More data needed.
- Ξ^- atoms: not observed yet. Favourable predictions. A proposal at J-PARC.

Acknowledgements

Thanks to my long-time collaborators
Chris J. Batty, Avraham Gal and Jiří Mareš.