Model independent form factor relations at large N_c

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based on T.D. Cohen, V. Krejčiřík, Phys. Rev. C **85** 035205 (2012)

introduction

- Theory of strong interaction — Quantum Chromodynamics
  - gauge theory of quarks and gluons based on $SU(N_c = 3)$ symmetry
- Practical problem — QCD is strongly coupled at low energies
  - conventional perturbative expansion is not applicable
  - expansion around non-interacting theory
  - corrections in the powers of coupling constant

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- Practical problem — QCD is strongly coupled at low energies
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  - expansion around non-interacting theory
  - corrections in the powers of coupling constant
- Some useful approaches
  - expansion around large-N_c limit
  - expansion around massless-quark (chiral) limit

introduction — two limits of QCD

- Large N_c world
  - number of colors N_c is a hidden free parameter of QCD ($SU(N_c)$ gauge theory)
  - simplifies substantially in the limit $N_c \to \infty$ due to combinatorics properties of diagrams
- Chiral world
  - QCD possesses a new symmetry if quark masses are zero — chiral symmetry
  - new symmetry makes the problem simpler

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- Promising idea — develop models of QCD in these two limits
  - even though these limits do not completely describe the real world, they are believed to capture many of its (at least qualitative) details
  - systematic procedure how to include corrections in the powers of m_π and/or $1/N_c$
  - double limit is not uniform and ordering of limits does matter for certain observables

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  - large N_c — encoded in the very core of the models, in the semiclassical treatment
  - chiral — imposed later as a constraint on the dynamic of meson fields
  - models based on large N_c and chiral limits of QCD with $N_c \to \infty$ taken first

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- Holographic models based on gauge-gravity duality[2,3]
  - attracted wide interest recently
  - large N_c — encoded in the very core of the models ($N_c \to \infty$ taken first)
  - looks totally different (if nothing else they are formulated in five dimensions)

(1) T.H.R. Skyrme, Proc. Roy. Soc. Lond. A **260** (1961) 127.

(2) A. Pomarol, A. Wulzer, JHEP **03** (2008) 051.

(3) T. Sakai, S. Sugimoto, Prog. Theor. Phys. **113** (2005) 843.

introduction — baryon models

- Important to check, if large N_c and chiral physics are encoded correctly
  - of course, there is more to modeling QCD than getting large N_c and chiral behavior right
  - since there is, in principle, infinite number of models, it is important to have a simple pass/fail test

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- Model-independent relations
  - large set of large N_c consistency relations that constrain the longest distance behavior of the system
  - typically they fix how quantities diverge as $m_\pi \to 0$
  - unusable for many of the holographic models, since they have been done for $m_\pi = 0$
  - the need for conceptually new model-independent relation

model-independent relation

- New model-independent relation
  - use position-space electric and magnetic form factors (Fourier transforms of standard momentum-space ones[4])
  - finite and well defined even if $m_\pi = 0$

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$$\lim_{r\to\infty} \frac{r^2\, \widetilde{G}_E^{I=0}\, \widetilde{G}_E^{I=1}}{\widetilde{G}_M^{I=0}\, \widetilde{G}_M^{I=1}} = 18$$

- isoscalar electric $\widetilde{G}_E^{I=0}$
- isoscalar magnetic $\widetilde{G}_M^{I=0}$
- isovector electric $\widetilde{G}_E^{I=1}$
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model-independent relation

- The relation was originally derived in the context of chiral soliton models[5]
- The question of model-independence arises
  - plausible to believe so
    - does NOT depend on any details of the model
    - in the past, all such relations derived in the chiral soliton models turned out (after deeper investigation) to be model independent
  - the purpose of this work is to prove the relation in a model-independent way

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- Not all models on the market satisfy it
  - something is wrong with Sakai-Sugimoto ("top-down") model
  - the underlying reason for this appears to be due to a failure of the flat-space instanton approximation(6)

(5) A. Cherman, T.D. Cohen, M. Nielsen, Phys. Rev. Lett. **103** (2009) 022001.

(6) A. Cherman, T.Ishii, arXiv:1109.4665v2[hep-th] (2011).

model-independent relation

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- Interesting properties
  - **all** low-energy constants, normalization of currents, sign and Fourier transform conventions **cancel**
  - universal number and power of r remain
  - calculable in a closed form for $m_\pi = 0$
  - does depend on the ordering of large N_c and chiral limits
- Model-independent calculation is done in the large N_c chiral perturbation theory

inputs of the calculation

- Key features of large N_c χPT
  - baryon mass is parametrically large (of order N_c) — heavy baryon approximation
  - pion loops contribute to the leading order — longest distance behavior is given by the currents connected to the pion loops with smallest possible number of pions
  - large N_c also eliminates diagrams suppressed by factor $1/N_c$
  - large N_c consistency relations implies that the Δ is degenerate with nucleon (generally whole tower of $I = J$ isobars) — Δ must be included in the calculation
    - mass difference $\Delta = M_\Delta - M_N$ is of order $1/N_c$ and serves as a new low energy constant
  - the form of pion-baryon-baryon′ vertex is determined by the large N_c consistency relations

inputs of the calculation

- Feynman rules for vertices
  - photon-two pions : $\epsilon_{a3b} A_\mu (p_a^\mu + p_b^\mu)$
    - key for isovector current, see ϵ_{a3b}
  - photon-three pions : $\frac{1}{12\pi^2 f_\pi^3} \epsilon_{abc} \epsilon^{\mu\nu\kappa\lambda} A_\mu p_{a\nu} p_{b\kappa} p_{c\lambda}$
    - key for isoscalar current, see ϵ_{abc}
  - pion-baryon-baryon′ : $\frac{g_A}{2f_\pi} \sqrt{\frac{2J^{(B')}+1}{2J^{(B)}+1}} \tau_a^{(BB')} \sigma_i^{(BB')} p_i$
    - determined by the consistency relations of large N_c QCD
    - matrices $\tau^{(BB')}$ ($\sigma^{(BB')}$) act in isospin (spin) space, they are a generalization of Pauli matrices, which appear in the pion-nucleon-nucleon vertex
- Feynman rules for propagators
  - pions : $\Delta^\pi(k) = \frac{i}{k^2 - m_\pi^2 + i\epsilon}$
    - fully relativistic propagators for pions, in the end $m_\pi \to 0$
  - baryons : $\Delta^N(k) = \frac{i}{k^0 + i\epsilon}$, $\Delta^\Delta(k) = \frac{i}{k^0 - \Delta + i\epsilon}$
    - non-relativistic propagators for baryons

inputs of the calculation

- Coupling matrices

$$\tau_1^{(NN)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2^{(NN)} = i\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \tau_3^{(NN)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\tau_1^{(N\Delta)} = \begin{pmatrix} -\sqrt{\frac{3}{2}} & 0 \\ 0 & -\sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} & 0 \\ 0 & \sqrt{\frac{3}{2}} \end{pmatrix}, \quad \tau_2^{(N\Delta)} = i\begin{pmatrix} \sqrt{\frac{3}{2}} & 0 \\ 0 & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} & 0 \\ 0 & \sqrt{\frac{3}{2}} \end{pmatrix}, \quad \tau_3^{(N\Delta)} = \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \\ 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}$$

$$\tau_1^{(\Delta N)} = \begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \end{pmatrix}, \quad \tau_2^{(\Delta N)} = i\begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{pmatrix}, \quad \tau_3^{(\Delta N)} = \ldots$$

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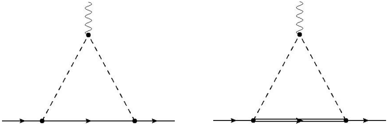
diagrams

- For isovector formfactors



  - two pion loop with either nucleon or delta in the intermediate state
  - totally two diagrams to be taken into account

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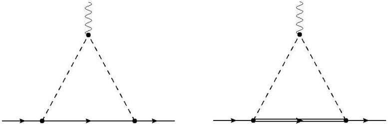

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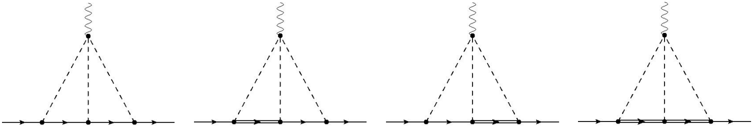
- For isoscalar formfactors



  - three pion loop with nucleons and deltas in the intermediate states
  - totally four diagrams to be taken into account

result

- Position-space form factors
  - evaluating diagrams, Fourier transforming, setting $m_\pi = 0$, extracting longest distance part :

$$\lim_{r\to\infty} \widetilde{G}_E^{I=0} = \frac{3^3}{2^9\pi^5}\,\frac{1}{f_\pi^3}\left(\frac{g_A}{f_\pi}\right)^3\frac{1}{r^9} \qquad \lim_{r\to\infty} \widetilde{G}_E^{I=1} = \frac{1}{2^4\pi^2}\left(\frac{g_A}{f_\pi}\right)^2\frac{\Delta}{r^4}$$

$$\lim_{r\to\infty} \widetilde{G}_M^{I=0} = \frac{3}{2^9\pi^5}\,\frac{1}{f_\pi^3}\left(\frac{g_A}{f_\pi}\right)^3\frac{\Delta}{r^7} \qquad \lim_{r\to\infty} \widetilde{G}_M^{I=1} = \frac{1}{2^5\pi^2}\left(\frac{g_A}{f_\pi}\right)^2\frac{1}{r^4}$$

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- Model-independent relation $\displaystyle \lim_{r\to\infty} \frac{r^2\,\widetilde{G}_E^{I=0}\,\widetilde{G}_E^{I=1}}{\widetilde{G}_M^{I=0}\,\widetilde{G}_M^{I=1}} = 18$ holds!
  - as advertised, all low-energy constants canceled

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  - cartoon picture (diagrams without Δ(s) + diagrams with Δ(s)

$$\approx\ e^{-m_\pi r} + e^{-m_\pi r}e^{-\Delta r}$$

limit $N_c \to \infty$

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limit $m_\pi \to 0$

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limit $r \to \infty$

$$\approx \; 1 + 0$$

limit $N_c \to \infty$

$$\approx \; 1 + 0$$

- $\lim\limits_{N_c \to \infty} \lim\limits_{r \to \infty} \lim\limits_{m_\pi \to 0} \dfrac{r^2 \, \widetilde{G}_E^{I=0} \, \widetilde{G}_E^{I=1}}{\widetilde{G}_M^{I=0} \, \widetilde{G}_M^{I=1}} = 9$

conclusion

- The relation $\lim_{r\to\infty} \frac{r^2\, \widetilde{G}_E^{I=0}\, \widetilde{G}_E^{I=1}}{\widetilde{G}_M^{I=0}\, \widetilde{G}_M^{I=1}} = 18$ was model-independently derived in the large N_c χPT
    - provided that the large N_c limit is taken at the outset of the problem

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  - provided that the large N_c limit is taken at the outset of the problem
- It may serve as an honest model-independent constrain on baryon models based on large N_c and chiral physics
  - it was shown to hold for:
    - Skyrme model[(5)]
    - ”bottom-up” holographic model[(5)]
    - ”top-down” holographic model (after rethinking induced by failing to satisfy new model-independent relation)[(6)]

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