Kosterlitz-Thouless type transition in Topologically Massive QED_3

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Abstract

Topologically Massive QED in (2+1) at T=0 is similar to XY model with finite chemical potential of vortex at finite-temperature and exhbits Kosterlitz-Thouless type transition.PTEP Febrary(2015)

0.1 Kosterlitz-Thouless transition (1973)

(2+1)-d boson model ; condensed phase

low temperature ;vortex pair are nuetral and there exists condensate

ϕ;phase fluctuation(continuous gauge transformation) ρ;uniform super fluid density,

$$\begin{aligned} \psi(x) &= \sqrt{\rho} e^{i\phi(x)}, \\ \left\langle \psi^{+}(x)\psi(0) \right\rangle &= \rho \left\langle e^{-i\phi(x)} e^{i\phi(0)} \right\rangle \\ &= \rho e^{-(D(0)-D(x-y))} \\ &= \rho \exp(-\frac{k_B T}{2\pi K_0} \ln(\frac{r}{a})) \propto r^{-\eta}. \end{aligned}$$

where we used

$$\begin{aligned} &\left\langle 0 | T(e^{-i\phi(x)} e^{i\phi(y)}) | 0 \right\rangle \\ &= -\frac{1}{2} \left\langle 0 | (\delta\phi^2(x) + \delta\phi^2(y) - 2T(\delta\phi(x)\delta\phi(y)) | 0 \right\rangle \\ &= -(D(0) - D(x-y)). \end{aligned}$$

a;short distance cut-off,$K_0 = (h/2\pi m)^2 \rho d, d = thickness.$

vortex;singular gauge transformation (discontinuous)

$$\phi(r) = \phi(z) = \pm \Im \ln(z - z_0).$$

high temperature;single vortex excitation lower the energy but unstable.

Free energy may be written

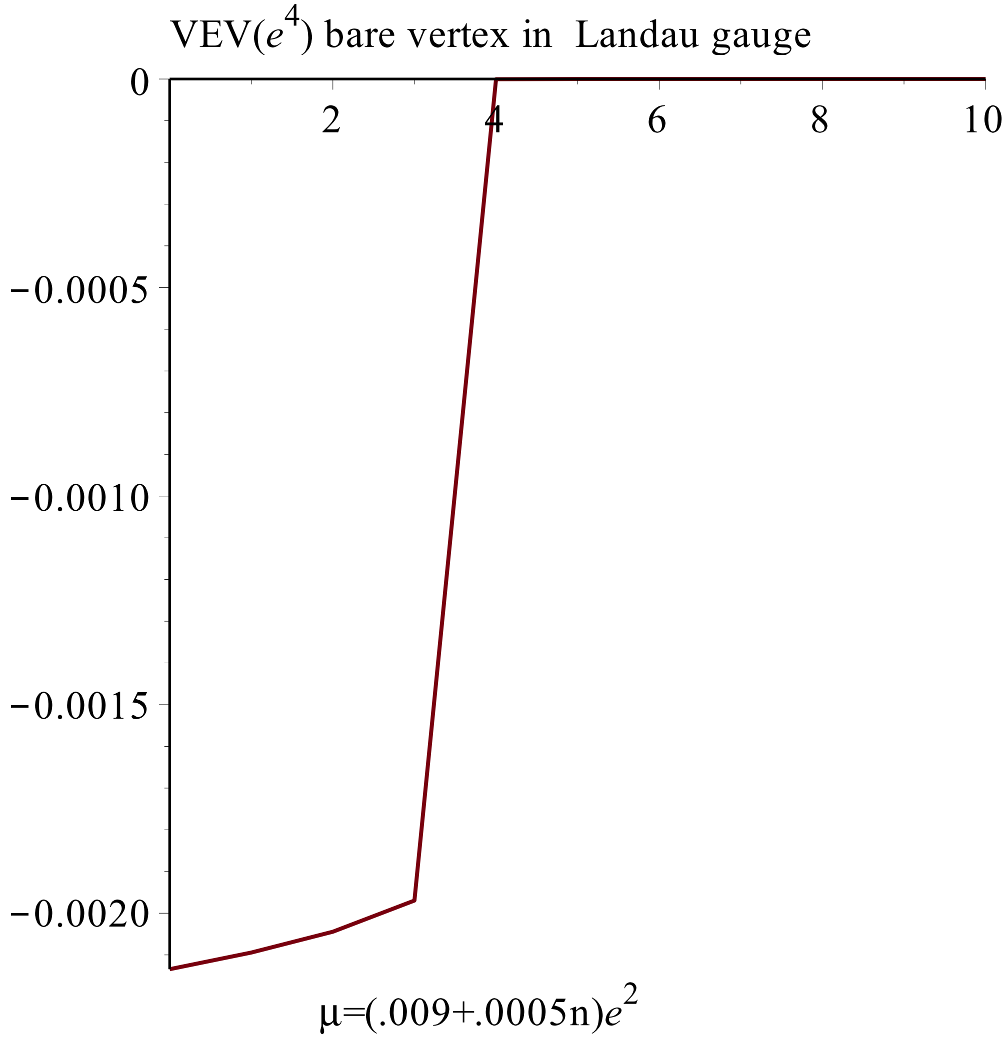
$$F_{SV} = E - TS = \pi K_0 \ln(\frac{L}{a}) - k_B T \ln(\frac{L}{a})^2. \quad (1)$$

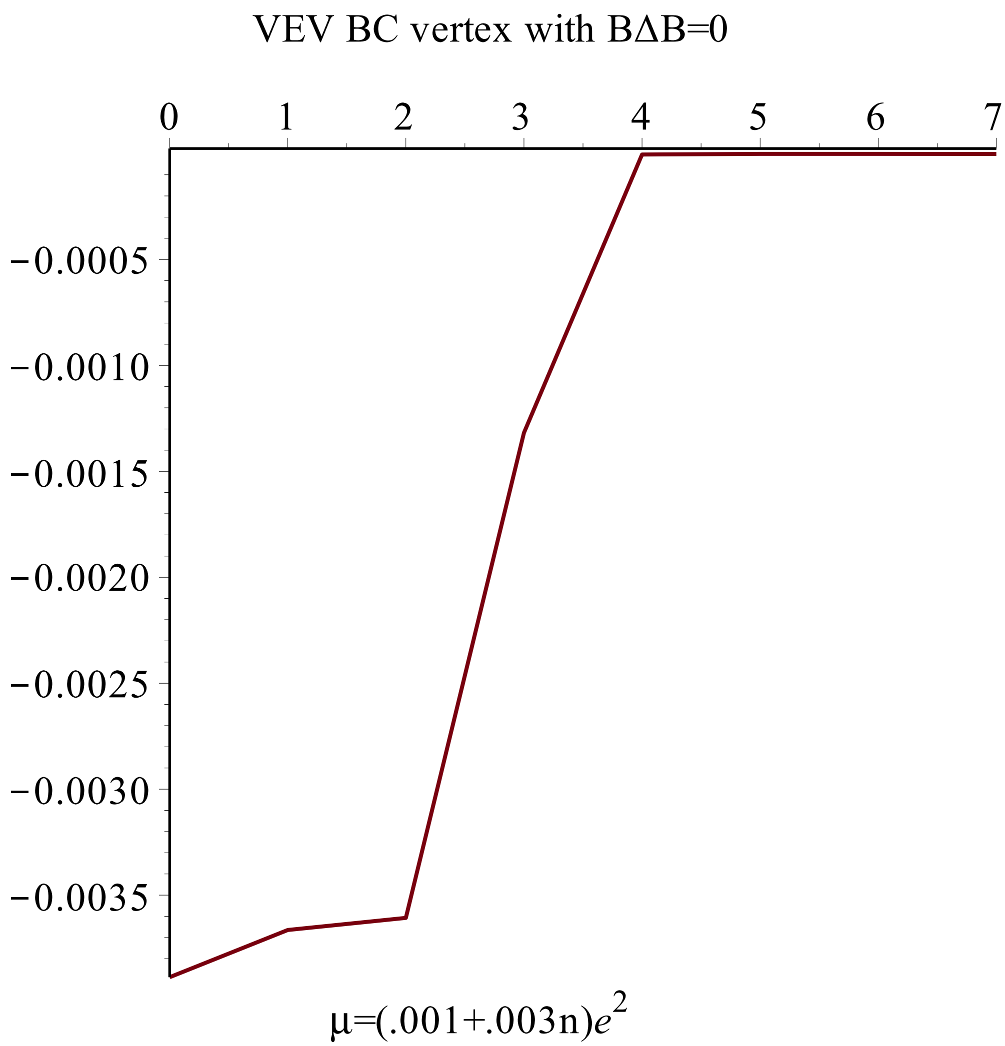
$$T \geq \frac{\pi K_0}{2k_B}, \quad (2)$$

free vortex may appear.From this fact critical temperature of super fluidity is determined

$$T_c = \frac{\pi K_0}{2k_B}. \quad (3)$$

$T_c = \pi K_0/2k_B, \eta \geq 1/4$,high temperature phase $\rho = 0$. Dynamics ???????


VEV(e^4) bare vertex in Landau gauge
0
2
4
6
8
10
−0.0005
−0.0010
−0.0015
−0.0020
μ=(.009+.0005n)e^2


VEV BC vertex with BΔB=0
0
1
2
3
4
5
6
7
−0.0005
−0.0010
−0.0015
−0.0020
−0.0025
−0.0030
−0.0035
μ=(.001+.003n)e^2

0.2 Topologically Massive QED$_3$ with 4-component fermion

S.Deser, R.Jackiw and S.Templeton(1982)

$$\mathcal{L} = \mathcal{L}_{QED} - \frac{\mu}{4}\epsilon^{\mu\nu\rho}A_\mu F_{\mu\nu}$$

$$\mathbf{A}(x)_{|x|\to\infty} \to \frac{-Q}{2\pi\mu}\nabla \arctan(\frac{y}{x}). \qquad (4)$$

There is a 2-spin degree of freedom.($\mu < 0, \mu > 0$):neutral parity conserving theory.(low temperature phase)

one degree:chiral (parity violating) theory,Topologically Massive QED.(high temperature phase)

Vortex is a degree of freedom of singular gauge transformation

$$\triangle_x \phi(x) = \delta^{(2)}(x), \phi(x) = \mu\pi \arctan(y/x) \qquad (5)$$

$\psi(r) \to \exp(i\phi(x))\psi(r)$,$\mu$ is related to Hall conductance.

0.3 symmetry

If the fermion is massless $m = 0$,$\mathcal{L}$ has $U(2)$ symmetry generated by $\{I, \gamma_3, \gamma_5, \tau\}$, $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$ for $\mu, \nu = (0, 1, 2)$, $\gamma_0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$, $\gamma_{1,2} = \begin{pmatrix} i\sigma_{1,2} & 0 \\ 0 & -i\sigma_{1,2} \end{pmatrix}$,

$\gamma_3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$, $\gamma_5 = \begin{pmatrix} 0 & -iI \\ iI & 0 \end{pmatrix}$,

$\tau = -i[\gamma_3, \gamma_5]/2 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$.γ_3 and γ_5 act as chiral transfomation

$$\begin{aligned} \psi(x) &\to e^{i\gamma_3\theta_3}\psi(x), \\ \psi(x) &\to e^{i\gamma_5\theta_5}\psi(x). \end{aligned} \tag{6}$$

Scalar density is mixed by chiral transformation

$$\begin{aligned} \overline{\psi}(x)\psi(x) \to \cos(2\theta_{3,5})\overline{\psi}(x)\psi(x) \\ + i\sin(2\theta_{3,5})\overline{\psi}(x)\gamma_{3,5}\psi(x), \end{aligned} \tag{7}$$

$$\overline{\psi}(x)\tau\psi(x) \to \overline{\psi}(x)\tau\psi(x). \tag{8}$$

Dynamical mass generation breaks $U(2)$ symmetry down to $U(1)_I \times U(1)_\tau$.

We find the eigenvalue of the free particle Hamiltonian

$$H = \gamma^0(\gamma^i p^i + m_e I + m_o \tau) \quad (9)$$

as $E^2 = p^2 + m_\pm^2, m_\pm = m_e \pm m_o$,where $i = 1, 2, I$ is a 4×4 unit matrix and τ is a operator defined above.Two kinds of mass are written by

$$m_e I + m_o \tau = \begin{pmatrix} m_+ & 0 \\ 0 & m_- \end{pmatrix} = \begin{pmatrix} m_e + m_o & 0 \\ 0 & m_e - m_o \end{pmatrix}. \quad (10)$$

So that we may split 4-component spinor into upper and lower components by projection operator

$$\psi_\pm = \chi_\pm \psi = \chi_\pm \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}, \chi_\pm = \frac{1 \pm \tau}{2}, \quad (11)$$

$$\chi_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \chi_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \quad (12)$$

Fom the Lagrangian

$$\mathcal{L} = \overline{\psi}_+(i\gamma \cdot \partial - m_+)\psi_+ + \overline{\psi}_-(i\gamma \cdot \partial - m_-)\psi_- \quad (13)$$

we have the free propagator

$$S(p) = -\frac{\gamma \cdot p + m_+}{p^2 - m_+^2}\chi_+ - \frac{\gamma \cdot p + m_-}{p^2 - m_-^2}\chi_-. \qquad (14)$$

0.4 Dyson-Schwinger equation

∘Chiral symmetry breaking order parameter $\left\langle \overline{\psi}\psi \right\rangle = -Tr(S_F(x)$

$\longrightarrow$ 0 by vortex effects at critical μ_{cr}.

∘Chiral symmetric phase with parity violating mass for $\mu \geq \mu_{cr}$.

$\mu_{cr} = 10^{-2}e^2$ by queched Landau gauge & full vertex correction with Ball-Chiu anzats.

$$(D^{-1})_{\mu\nu} = i(p^2 g_{\mu\nu} - p_\mu p_\nu + i\mu\epsilon_{\mu\nu\alpha}p^\alpha) + i\frac{p_\mu p_\nu}{\xi}. \qquad (15)$$

$$(D^{-1})_{\mu\nu} D_{\nu\rho} = g_{\mu\rho}, \tag{16}$$

$$\begin{aligned} D_{\mu\nu}(k) &= \frac{1}{i}[\frac{g_{\mu\nu} - k_\mu k_\nu/(k^2+i\epsilon) - i\mu\epsilon_{\mu\nu\rho}k^\rho/(k^2+i\epsilon)}{k^2-\mu^2+i\epsilon} \\ &\quad +\xi\frac{k_\mu k_\nu}{(k^2+i\epsilon)^2}]. \end{aligned} \tag{17}$$

The Schwinger-Dyson equation for the self-energy $\boldsymbol{\Sigma}(p)$ for BC vertex is written

$$\begin{aligned} -i\boldsymbol{\Sigma}(p) &= (-ie)^2 \int \frac{d^3q}{(2\pi)^3} \frac{\gamma_\mu(q\cdot\gamma A(q) + B(q))}{(q^2A^2(q) - B^2(q))} \\ &\quad \times \Gamma_\nu^{BC}(q,p) D_{\mu\nu}(q-p), \end{aligned} \tag{18}$$

$$\begin{aligned} \Gamma_\mu^{BC}(p,q) &= \Gamma_\mu^T(p,q) + \frac{A(p)+A(q)}{2}\gamma_\mu \\ &\quad + \frac{A(p)-A(q)}{2(p^2-q^2)}\gamma\cdot(p+q)(p+q)_\mu \\ &\quad - \frac{B(p)-B(q)}{p^2-q^2}(p+q)_\mu. \end{aligned} \tag{19}$$

W-T

$$(p-q)_\mu \Gamma_\mu^{BC}(p,q) = A(p)\gamma\cdot p - A(q)\gamma\cdot q - (B(p)-B(q)).$$

0.5 determination of the critical point by spectral function

S.Weinberg(1965),R.Jackiw&L.Solovief(1968),Y.Hoshino(2004)

$$\rho(x) = e^{F(\mu|x|)}, \quad (20)$$
$$S_F(x) = S_F^0(x)\rho(x). \quad (21)$$

F is a Fourier transformation of decay probability;$e(p) \rightarrow e(r) + gauge\ boson(k)$

$$F = \int \frac{d^3k}{(2\pi)^2}\theta(k_0)\delta(k^2)2k_0\frac{E_r}{m}\sum_{\lambda,S} T_1\overline{T_1}e^{-ik\cdot x}, \quad (22)$$

$$\begin{aligned}\rho(p) &= \frac{1}{\pi}\,\mathrm{Im}\, iS_F(p) = \gamma\cdot p\rho_1(p) + \rho_2(p)\\ &= (2\pi)^2\sum_n \delta^{(3)}(p - p_n)\left\langle 0|\psi(0)|n\right\rangle\left\langle n|\overline{\psi}(0)|0\right\rangle.\end{aligned} \quad (23)$$

$$T_1 = \langle 0|\psi(0)|r,k\rangle$$
$$= -i\left\langle 0^{in}|T[\psi(0), e\int d^3x\overline{\psi}^{in}(x)\gamma_\mu\psi^{in}(x)A^{in}_\mu(x)]|r;k \text{ in}\right\rangle \tag{24}$$
$$= -ie\int d^3xS^0_F(0-x)\gamma_\mu\langle 0|\psi(x)|r\rangle\langle 0|A_\mu(x)|k\rangle$$
$$= \frac{-ie}{\gamma\cdot(r+k)-m+i\epsilon}\gamma_\mu\epsilon^\mu_\lambda(k)U_S(r)\sqrt{\frac{m}{E_r}}\frac{1}{\sqrt{2k_0}}, \tag{25}$$

In 4-dim we have

$$S(p) \sim (\gamma\cdot p+m)(p^2-m^2)^{-1+\beta},$$
$$\beta = e^2(\xi-3)/8\pi^2.$$

In 3-d Minkowski space;we have infrared logarithm and cut & dipole$\times$ **log** type singularity.So that the high energy behaviour is dominated by single particle singularity with log correction.However if we exponentiate this log correction it turns out to be a power like singularity.This term dominate the high-energy behaviour and we obtain condensation and finite value of the order parameter$\left\langle\overline{\psi}\psi\right\rangle$ .In position space it is easy to see,because no ultraviolet divergences.

$$\begin{aligned} S_F^0 &= -(i\gamma\cdot\partial + m)\frac{e^{-mr}}{4\pi r}, \\ S_F^{'} &= -(i\gamma\cdot\partial + m)Ce^{-mr}(mr \ll 1). \end{aligned} \quad (26)$$

mC : $O(10^{-3}e^4)$Only short distance behaviour is modified.For $\mu \neq 0$ case ,we evaluate F for chiral representation S_+ and get gauge invariant

$$F(\mu|x|) = (-\frac{\gamma\cdot r + m}{2m}\frac{e^2}{8\pi m} + \frac{\gamma\cdot r}{m}\frac{e^2}{32\pi\mu} - \frac{e^2\mu}{32\pi m^2})E_1(\mu|x|),$$

$$\begin{aligned} &E_1(z) = \int_z^\infty \frac{e^{-t}}{t}dt, (|\arg z| < \pi) \\ &E_1(x) \sim -\gamma - \ln(x), x \ll 1, \end{aligned}$$

$$\begin{aligned} e^F &= \frac{\gamma\cdot r + m}{2m}(\mu|x|)^D \exp(\gamma)\frac{e^2\mu}{32\pi m^2}\ln(\mu|x|)), \\ D &= \frac{e^2}{8\pi m} - \frac{e^2}{32\pi\mu}, \end{aligned} \quad (27)$$

If we set wave renormalization $D = 0, m = e^2/8\pi, \mu = e^2/32\pi$

$$\begin{aligned} e^F &= \frac{\gamma \cdot r + m}{2m} e^{\gamma} \frac{1}{16} \ln(\frac{e^2}{32\pi}|x|) \\ &\simeq \frac{\gamma \cdot r + m}{2m} .1 \ln(.01 e^2 |x|) \end{aligned}$$

$$\int_0^{\infty} \frac{e^{-mr}}{4\pi r} \frac{\sin(pr)}{pr} E_1(\mu r) r^2 dr \tag{28}$$

$$= \frac{-1}{2(p^2+m^2)} \ln(\frac{p^2+m^2}{\mu^2}) + \frac{m}{(p^2+m^2)p} \arctan(\frac{p}{m}),$$

Only parity Violating mass $m \propto 1/p$ is generated. $\left\langle \overline{\psi} \tau \psi \right\rangle \propto \ln(\Lambda/m)$

0.6 Summary

We studied the effects of vortex on the chiral condensate by Dyson-Schwinger equation of the self-energy of fermion and the spectral function of the propagator in Topologically Massive QED_3.These show the detaild dynamics of vortex on condensed matter physics which has not been clear.Especially in the spectral function we find how the vortex washes away the condensate in terms of anomalous dimension.In the BC vertex case we do not find the clear signal of first-order phase transition.We find the oscillation of $\left\langle \overline{\psi}\psi \right\rangle$ with μ in by $B\Delta B$ in A(p).If we neglect this term,there may be a first order phase transition.Our results may be related to $T_{KT} = 0$ case with no super conductivity or super fluidity.

K.I.Kondo &P.Maris(1996),A.Raya et.al(2011),

Y.Hoshino,T.Inagaki,Y.Mizutani

argued 1-st order phase transition.Physical meanings has not been clear.