Torsional Instantons In Quantum Gravity: Correspondence with condensed matter systems?

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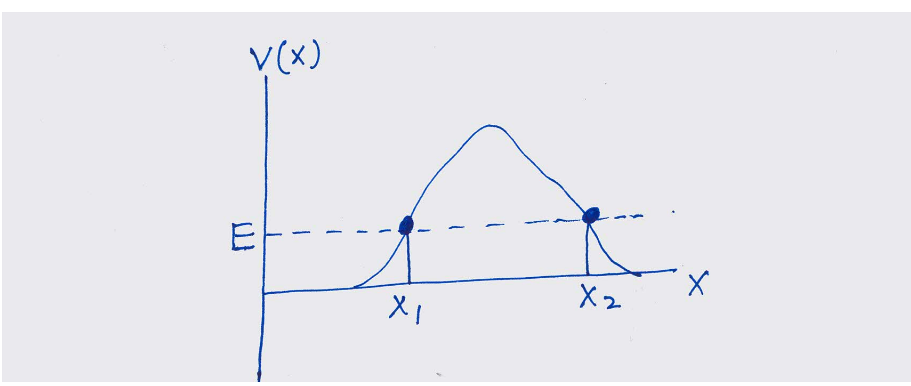
[References:
R.Kaul and S.Sengupta, PRD90, 124081, 2014;
S.Sengupta, CQG32, 195005, 2015]

Instantons

• Instantons are solutions of classical equations of motion in Euclidean time ($t \to it = \tau$), having finite action

• Typically, instantons are used as a tool to explain certain nonperturbative phenomena in QM and QFT (processes whose amplitudes don't admit a good power series expansion in h)

• For example, tunneling can be interpreted as an instanton effect; WKB formula for tunneling amplitude: $T(E) = e^{-\frac{1}{\hbar}\int_{x_1}^{x_2} dx \ [V(x) - E]^{\frac{1}{2}}}$



Instantons $\equiv$ Topological densities

- Instantons carry topological charge

- Existence of instanton effects necessarily implies that the corresponding classical Lagrangian is ambiguous upto the addition of topological densities:

$$L_{eff} \;=\; L_0 + \theta L_{Top} \;=\; L_0 + \theta\ \partial_\mu J^\mu_{Top}$$

- The topological densities, being total divergences, do not affect the classical dynamics

- However, quantum theory is affected, where the states and observables depend on θ, which appears as a quantum coupling constant (e.g. QM in Periodic potential, θ vacuum in gauge theories)

Instanton physics and quantum gravity

• Instanton methods can be particularly useful for analysing nonperturbative properties of (quantum) gravity theory in 4D

• In 1st order gravity (4D), there are three possible topological densities $I_E,\ I_P,\ I_{NY}$:
Euler: $\epsilon^{\mu\nu\alpha\beta}\epsilon_{IJKL}R_{\mu\nu}{}^{IJ}(\omega)R_{\alpha\beta}{}^{KL}(\omega)$
Pontryagin: $\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu}{}^{IJ}(\omega)R_{\alpha\beta IJ}(\omega)$
Nieh-Yan: $\epsilon^{\mu\nu\alpha\beta}\left[e^I_\mu e^J_\nu R_{\alpha\beta IJ}(\omega) - 2\left(D_\mu e^I_\nu\right)\left(D_\alpha e_{\beta I}\right)\right]$

• Most general gravity Lagrangian (with or without matter): $L = L_0 + \phi I_E + \theta I_P + \eta I_{NY}$

• Instantons carrying nontrivial Euler and Pontryagin topological charges are well-known

• But so far, no known example of a Nieh-Yan instanton (one that solves Euclidean EOM and carries a NY charge)

[Regge, D'Auria, Hanson, Eguchi, Tseytlin, Chandia, Zanelli,..]

Nieh-Yan instantons

$L = L_0 + ... + \eta I_{NY}$

- $I_{NY} = d(e \wedge T) = 0$ if torsion T is zero

- Thus, the natural arena to look for NY instantons is first order gravity theory (with or without matter) where torsion does not vanish apriori

- Is there a first order theory of gravity which has NY instantons?

- Do they lead to nonperturbative effects?

- Possible correspondence between the spacetime and (topologically nontrivial) condensed matter systems?

Gravity coupled to axionic gravity

- Consider a first order gravity theory coupled to antisymmetric tensor gauge field $B_{\mu\nu}$ (axion) since such (bosonic) matter can induce torsion in the first order theory

- Euclidean Lagrangian density:

$$L(e, \omega, B) = -\frac{1}{2\kappa^2} e e_I^\mu e_J^\nu R_{\mu\nu}^{\ \ IJ} + \beta e H^{\mu\nu\alpha} H_{\mu\nu\alpha} + \frac{1}{2\kappa} e H^{\mu\nu\alpha} e_\mu^I D_\nu e_{\alpha I}$$

where $H_{\mu\nu\alpha} = \partial_\mu B_{\nu\alpha} + \partial_\nu B_{\alpha\mu} + \partial_\alpha B_{\mu\nu} = \partial_{[\mu} B_{\nu\alpha]}$

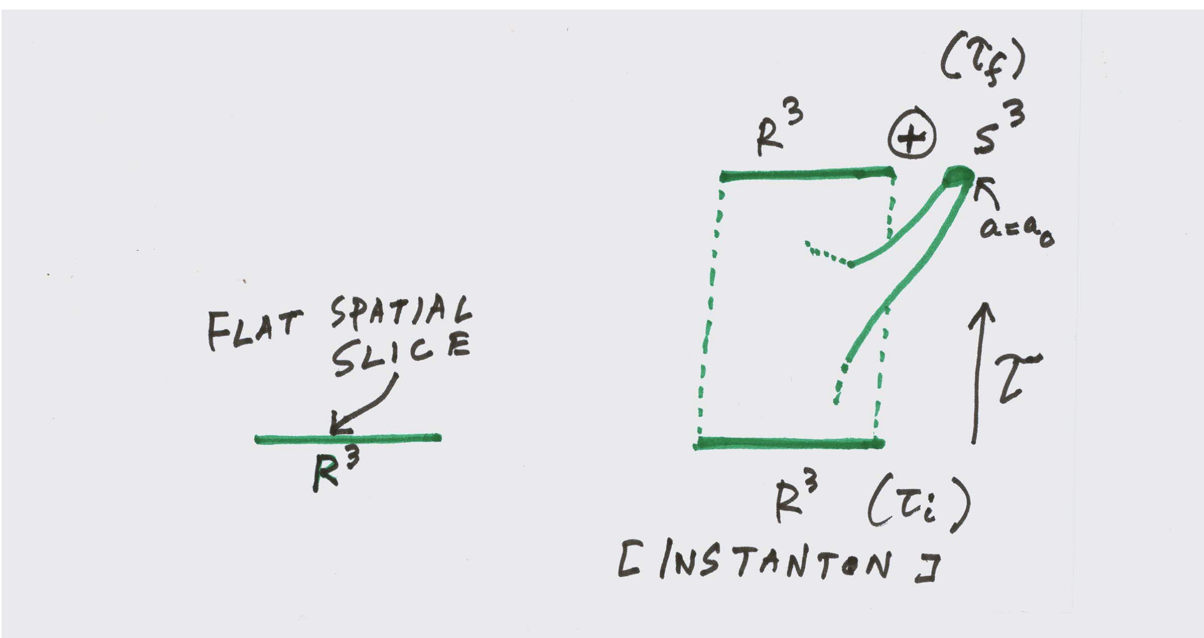
- The last term introduces a nonvanishing torsion (ω EOM): $T_{\mu\nu}^{\ \ I} := D_{[\mu} e_{\nu]}^I = \kappa H_{\mu\nu\alpha} e^{\alpha I}$

- Take a spherically sym metric
$ds^2 = d\tau^2 + a^2(\tau)\, d\Omega^2(\chi, \theta, \phi)$

Giddings-Strominger instanton

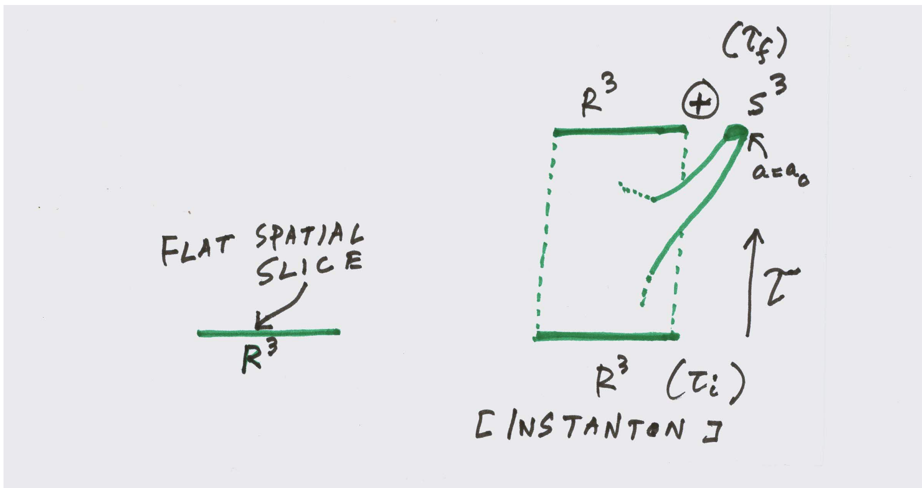
- The EOM for $a(\tau)$ becomes: $\dot{a}^2(\tau) = 1 - \frac{a_0^4}{a^4(\tau)}$

- $a(\tau)$ has a minimum; Can start at a very large value (flat 3-slice) and end at a_0 (3-sphere), or vice versa-Wormhole
(first discovered by Giddings-Strominger (1988) in second order axionic gravity)



- Each such instanton creates a baby universe (S^3) of axion charge $Q = \int_{S_3} d^3x \; \epsilon^{abc} H_{abc}$

GS Wormhole $\equiv$ NY instanton



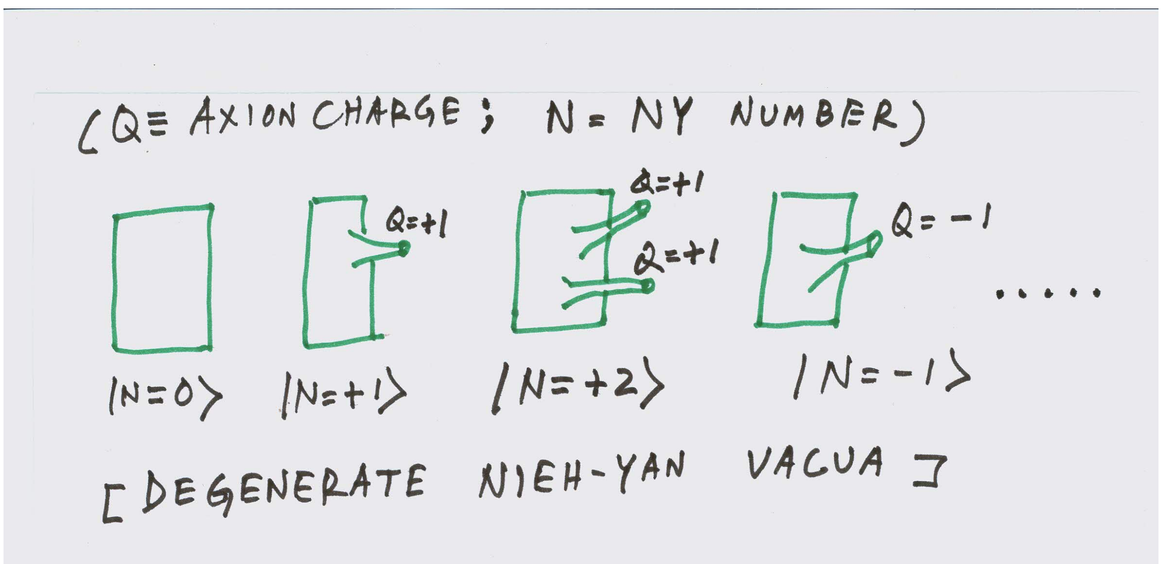
- And remarkably, these correspond to a non-vanishing torsional Nieh-Yan number:

$$N_{NY} = -\frac{1}{\pi^2\kappa^2}\int_{M^4} d^4x\ \partial_\mu\left[\epsilon^{\mu\nu\alpha\beta}e_{\nu I}D_\alpha(\omega)e^I_\beta\right]$$
$$= \frac{1}{2\pi^2\kappa}\int_{S^3} d^3x\ [\epsilon^{abc}H_{abc}] = Q$$

- We have found the elusive gravitational instanton with NY topological charge-GS wormholes!

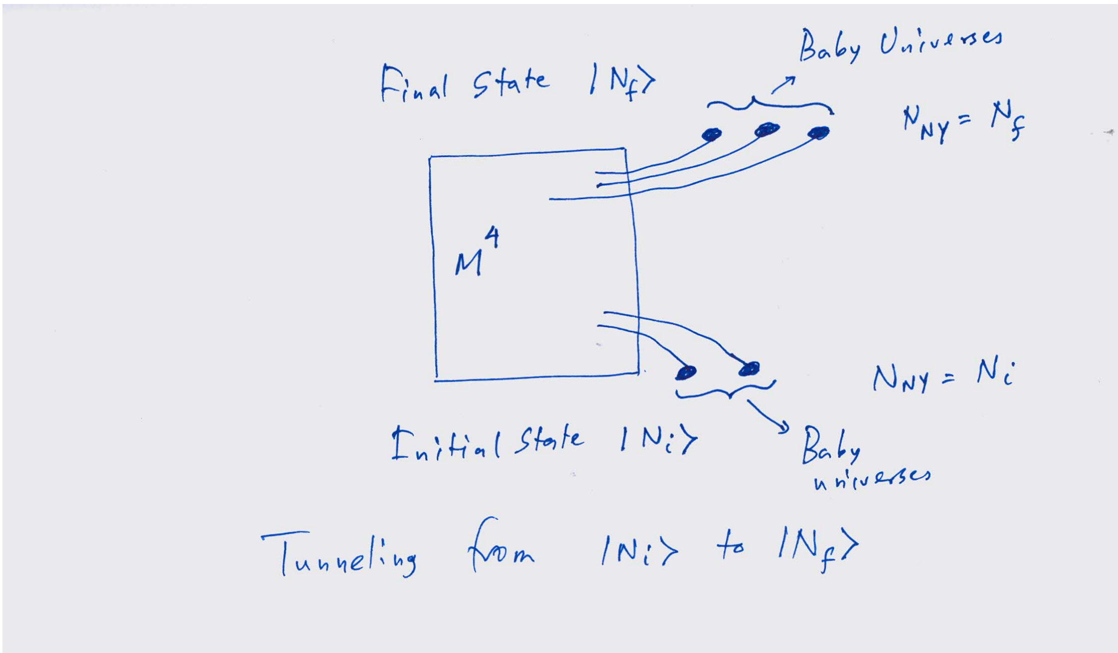
Tunneling: Nonperturbative effects

• Naively, there can be infinite number of ground states with different Nieh-Yan numbers N (baby universes): degenerate perturbative vacua $|N\rangle$



• However, NY instantons induce tunneling between these states and break this degeneracy

η Vacuum in quantum gravity



- As in QCD, the true vacuum of quantum gravity is a nonperturbative one: $|\eta\rangle = \sum_N e^{i\eta N}|N\rangle$, characterised by a new coupling constant η

- The Hamiltonian or the 'vacuum energy density' receives a modification of the size:

$$\rho_\eta = -2e^{-S_{inst}} K cos\eta$$

[See PRD 2014, R. Kaul and S.S for details]

To summarize..

- There exist torsional instantons (GS wormholes) in first order gravity, which induce tunneling effects; This leads to a nonperturbative ground state $|\eta\rangle$ in quantum gravity

- This vacuum is parametrised by a P and T odd 'quantum' coupling constant η (Barbero-Immirzi parameter of loop quantum gravity), an exact analogue of the θ parameter of PP or QCD; Has to be fixed by experiments

Implications of $|\eta\rangle$ vacuum

• Such torsional instanton effects could be particularly relevant in the context of various parity violating phenomena in particle physics and cosmology

• Can the vanishingly small energy density of the η vacuum provide a possible solution to the cosmological constant problem?
[S.Sengupta, CQG32, 195005, 2015]

Correspondence between gravity and condensed matter

• There are many phenomena in condensed matter physics whose origin is nontrivial topology; e.g. Integer Quantum Hall effect
IQHE: $k \to H(k)$ (Pontryagin numbers)
QG: $x \to H_{\mu\nu\alpha}(x)$ (Nieh-Yan numbers/Axion charge)

• How are the torsional Nieh-Yan numbers perceived by the con-mat systems?

• In pure gravity also, there are torsional solutions (monopoles, instantons,..) with nontrivial NY topological number-Degenerate metric [In progress]

• Can one construct a sensible dynamical theory of gravity using such configurations? How far can one go using the resemblance of spacetime with crystals with defects (dislocations)? [In progress]

THANK YOU