Shell model calculation of isospin-breaking correction to superallowed Fermi beta-decay

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Outline

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  - The tests via superallowed $0^+ \to 0^+$ Fermi beta decay.
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  - Shell model approach
  - Radial overlap correction.
  - Woods-Saxon
  - Hartree-Fock (Skyrme-type interaction)
- Summary and perspectives

Motivation

Low energy tests of the Standard Model :

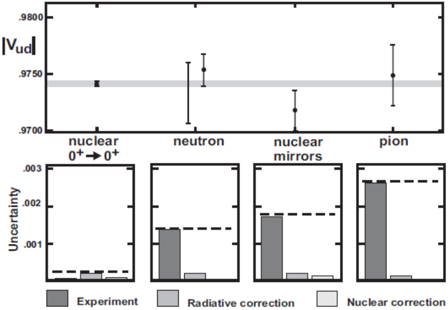
- CVC hypothesis : G_V = universal constant
  - The test of CVC is based on measurements of G_V in many $\beta-$decay processes
- Unitarity of the CKM matrix
  - Mixing between mass and weak eigenstates :

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

  - The model itself doesn't give numerical values for matrix elements, but it requires that $V^\dagger V = \mathbb{1}$.
    The most dominant element, $|V_{ud}| = G_V/G_\mu$ can be obtained from nuclear physics studies.

Motivation

- Current status of $|V_{ud}|$ (Hardy-Towner, PRC 91, 025501, 2015)
  - Four different weak processes have been considered
  - Experimental inputs : Q_{EC}, $t_{1/2}$, BR and $\lambda \sim GT/F$ (for axial-vector)



  - Superallowed $0^+ \rightarrow 0^+$ beta decay (nuclear structure)
  - Decay of free neutron (GT/F)
  - Mirror transition (GT/F and nuclear structure)
  - Decay of pion (very weak branching ratio $\sim 10^{-8}$)

Motivation

$|V_{ud}|$ from superallowed $0^+ \rightarrow 0^+$ Fermi beta decay

- Selection rules : $\Delta J = 0$, $\Delta\pi = NON$, $\Delta T = 0$, thus, these transitions can only occur between isobaric analogue states. ISB effects causes a reduction in $|M_F|^2 = 2(1 - \delta_C)$.

Basic weak-decay equation :

$$Ft = ft(1 + \delta_R')(1 - \delta_C + \delta_{NS}) = \frac{K}{2G_V^2(1 + \Delta_R^V)}$$

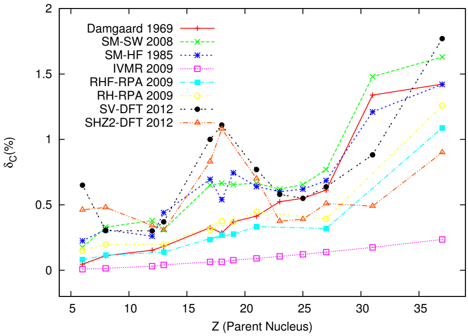
- f, statistical rate function, $f(Z, Q_{EC})$
- t, partial half-life $= t_{1/2}/BR$
- δ_C, isospin-breaking correction

- Radiative corrections : δ_R'= nuclear structure independent, δ_{NS}= nuclear structure dependent, Δ_R^V= transition independent

For 14 cases (^{10}C to ^{74}Rb), ft has been measured with precision $\leq 0.1\%$, this study is now limited by δ_C.

Motivation

- Isospin correction, δ_C (1969-present)



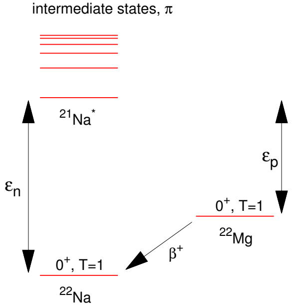
- HO (Damgaard)
- SM-WS (Towner and Hardy)
- SM-HF (Ormand and Brown)
- RH, RHF-RPA (Liang et al.)
- SV, SHZ2-DFT (Satula et al.)
- IVMR (Auerbach)

- δ_C is strongly model dependent
- Shell-model results (SM-WS and SM-HF) agree well with CVC, but they are in overall disagreement in magnitude.

Shell model calculation

- Fermi matrix element (in parentage expansion formalism) :

$$M_F = \langle f|\,\tau_+\,|i\rangle = \sum_{\alpha,\pi} \langle \alpha|\bar{\alpha}\rangle^{\pi} \langle f|a_{\alpha}^{\dagger}|\pi\rangle \langle \pi|a_{\bar{\alpha}}|i\rangle$$



- $\langle \alpha|\bar{\alpha}\rangle^{\pi}$ can be calculated with eigenstates of a realistic potential
- Ideally, $\langle f|a_{\alpha}^{\dagger}|\pi\rangle$ and $\langle \pi|a_{\bar{\alpha}}|i\rangle$ can be obtained with SM + INC effective interactions, but states beyond model space contribute significantly.

Shell model calculation

- Towner-Hardy's method : Isospin-symmetry breaking occurs in two ways :
  - Radial integrals depart from unity, $\langle\alpha|\bar{\alpha}\rangle^{\pi} \neq 1$:

$$\sum_{\alpha,\pi} \langle\alpha|\bar{\alpha}\rangle^{\pi} \left|\langle f|a_{\alpha}^{\dagger}|\pi\rangle\right|^{2} = M_{F}^{0}(1-\delta_{RO})^{1/2}$$

  - The spectroscopic amplitudes do not satisfy hermiticity because of INC terms in the shell-model hamiltonian, $\langle\pi|a_{\bar{\alpha}}|i\rangle \neq \langle f|a_{\alpha}^{\dagger}|\pi\rangle^{*}$:

$$\sum_{\alpha,\pi} \langle f|a_{\alpha}^{\dagger}|\pi\rangle \langle\pi|a_{\bar{\alpha}}|i\rangle = M_{F}^{0}(1-\delta_{IM})^{1/2}$$

Where $\delta_{C} \approx \delta_{RO} + \delta_{IM}$

Shell model calculation

- Basic ingredients for radial overlap correction, δ_{RO}
  - Spectroscopic amplitudes :

$$\langle f|a_\alpha^\dagger|\pi\rangle \quad \text{or} \quad \langle \pi|a_\alpha^\dagger|i\rangle$$

    Using large-scale shell-model calculations with well established effective interactions : USD/USDA/USDB for $22 \leq A \leq 38$, KB3G/GXPF1A for $46 \leq A \leq 54$, and JUN45 for $A = 62$ and 66.

  - Radial integrals :

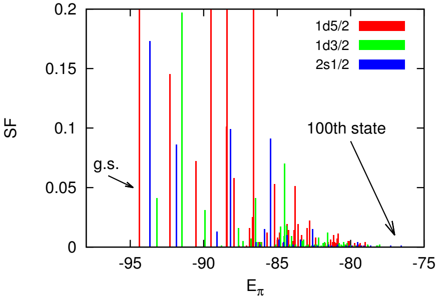
$$\langle \alpha|\bar{\alpha}\rangle^\pi = \int_0^\infty R_\alpha^\pi(r) R_{\bar{\alpha}}^\pi(r) r^2 dr$$

    WS or Skyrme-HF radial wave functions, constrained by reproduction of separation energies and/or charge radius

δ_{RO} requires a large number of parent states π.

Shell model calculation

- Cut-off for δ_{RO} (e.i. neutron pick-up : ^{26}Mg $\rightarrow$ ^{25}Mg using USD interaction)



- Sum rules are also very well satisfied

orbit	$\sum SF$	$\langle n_\nu \rangle$
$\nu 1d5/2$	4.793	4.82
$\nu 2s1/2$	0.557	0.56
$\nu 1d3/2$	0.613	0.62

- First 100 states are enough for nuclei under consideration ($22 \leq A \leq 66$).

- We have succeeded in diagonalizing without truncation for 13 cases : *sd* (^{22}Mg, ^{26}Al, ^{26}Si, ^{30}S, ^{34}Cl, ^{34}Ar, ^{38}K, ^{38}Ca), *fp* (^{46}V, ^{50}Mn, ^{54}Co) and $f5pg9$ (^{62}Ga, ^{66}As)

Shell model calculation

- Shell-model configuration spaces

Parent nuclei	This work	Towner-Hardy(2002)	Ormand-Brown(1985)
$22 \leq A \leq 38$	full *sd*	full *sd*	full *sd*
^{46}V	full *fp*	full *fp*	full *fp*
^{50}Mn	full *fp*	$(f7)^{10-r}(f5p)^{r}$ *	$(f7p3)^{10}(f7)^{n_7}(f5)^{n_5}(p1)^{n_1}$ †
^{54}Co	full *fp*	$(f7)^{14-r}(f5p)^{r}$	$(f7p3)^{14}(f7)^{n_7}(p3)^{n_3}(f5)^{n_5}(p1)^{n_1}$ ‡
^{62}Ga	full $f5pg9$	$(f7)^{16}(f5p)^{6}$	$(f7)^{16}(f5p)^{6}$
^{66}As	full $f5pg9$	$(f7)^{16}(f5p)^{10}$	$(f7)^{16}(f5p)^{10}$

*. $r \leq 2$

†. $n_5 + n_1 = 1$

‡. $n_3 + n_5 + n_1 = 2$

Shell model calculation

- Calculation with WS radial functions

$$V(r) = -Vf(r, a_0, r_0) - V_{so}\frac{1}{r}\frac{d}{dr}f(r, a_s, r_s)\langle \boldsymbol{l}.\boldsymbol{\sigma}\rangle + V_{coul}(r)$$

	Bohr-Mottelson (BM_m)	Schwierz-Wiedenhöver-Volya (SWV)
r_s	1.16	1.16
r_0	1.26	1.26
V_0	52.833	52.06
$a_0 = a_s$	0.662	0.662
V_1	146.368	-
V_{ls}	0.22	-
κ	-	0.639
λ	-	-24.1
V_{so}	$V_{ls} r_s^2 V$	$V_0 \lambda \hbar^2 / 4\mu^2 c^2$
V	$V_0 \pm V_1(N-Z)/4A$ §	$V_0(1 - 4\kappa \langle \mathbf{t}.\mathbf{T}' \rangle / A)$ ¶
V_{coul}	uniformly charged sphere	uniformly charged sphere

§. Symmetry term

¶. Isospin coupling of Lane : Nucl. Phys. 35, 676 (1962).

Shell model calculation

- Parametrization optimization
  - Well depth V_0 is readjusted to reproduce separation energies :

$$\epsilon_p = S_p + E_x(A-1) \quad \text{and} \quad \epsilon_n = S_n + E_x(A-1)$$

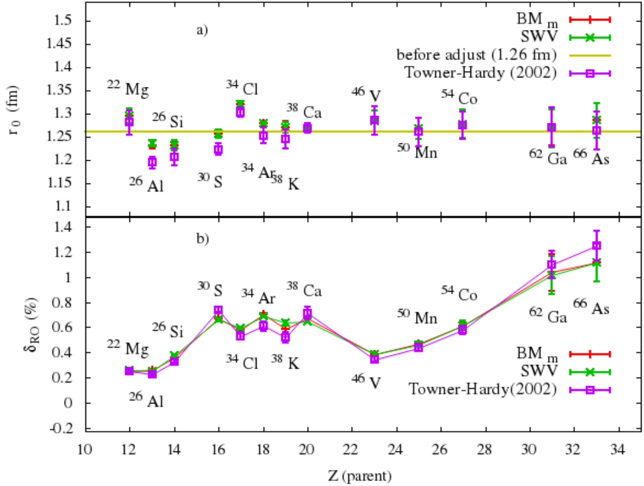
this is a strong contraint to get correct radial functions at large r : $R(r) \propto \exp(-\frac{\sqrt{2m\epsilon}}{\hbar} r)$.

  - Parameter r_0 is readjusted to fix charge radius of parent nucleus :

$$\langle r^2 \rangle_{ch} = \frac{1}{Z} \sum_{\pi\alpha} S(i\pi k_{\bar{\alpha}}) \int_0^\infty r^4 |R^{\pi}_{\bar{\alpha}}(r)|^2 dr + \left(\frac{3}{2} a_p^2 - \frac{3}{2} \frac{b^2}{A} \right)$$

occupation numbers are replaced by proton-SF, thus one can allow radial functions to depend on parent states

Shell model calculation



Showing good agreement with the work of Towner-Hardy, except for a few cases :

- For ^{34}Ar and ^{38}K, we used an update data of charge radius
- Reduction in ^{62}Ga and ^{66}As is caused by inclusion of $g9/2$

Shell model calculation

- Calculation with HF radial functions (with a Skyrme-type interaction)

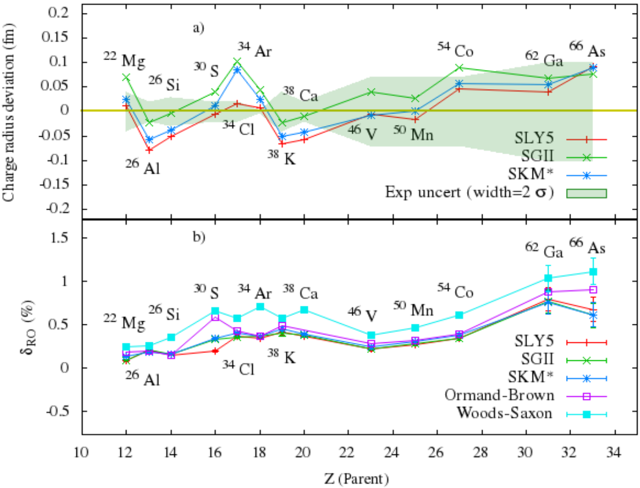
$$v_{Sk} = t_0(1 + x_0 P_\sigma)\delta + \frac{1}{2} t_1(1 + x_1 P_\sigma)(\mathbf{k}'^2\delta + \delta\mathbf{k}^2) + t_2(1 + x_2 P_\sigma)\mathbf{k}'.\delta\mathbf{k}$$
$$+ \frac{1}{6} t_3(1 + x_3 P_\sigma)\rho^\alpha(\mathbf{R})\delta + iW_0(\sigma_i + \sigma_j).\mathbf{k}' \times \delta\mathbf{k} + v_{coul}$$

  - Unlike the WS case, the Skyrme-HF field is nonlocal and there is no parameter that control potential shape.
  - Using $R^L_\alpha(r) = [m/m^*]^{1/2} R_\alpha(r)$ to obtain local equivalent potential :

$$V^L(r, \epsilon_\alpha) = V^0(r, \epsilon_\alpha) + V^{so}(r)\langle \mathbf{l}.\sigma \rangle + V_{coul}(r)$$

then, we can adjust by scaling $V^0(r, \epsilon_\alpha)$ to fix separation energies

Shell model calculation

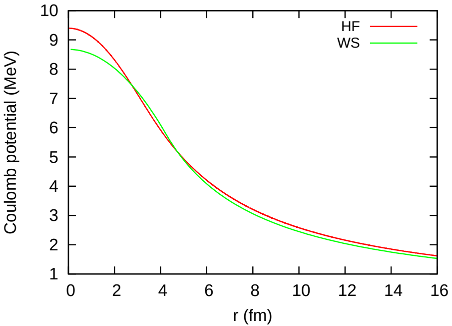


In ^{30}S, SLY5 gives smallest value because of J^2 term, OB got largest value because they used a resticted model space. The disagreement in ^{62}Ga and ^{66}As is due to $g9/2$.

Systematic reduction from WS results.

- There is a significant discrepancy between WS and HF at large r.

$$V_C(r \to \infty) = \begin{cases} \dfrac{Ze^2}{r} & \text{HF} \\ \dfrac{(Z-1)e^2}{r} & \text{WS} \end{cases}$$



- HF procedure is formally correct, but the Skyrme forces doesn't contain any ISB term.
- WS is phenomenologic and misses exchange terms

- δ_{RO} have been reexamined for 13 cases. Radial wave functions are obtained from WS and Skyrme-HF calculations. Shell-model input informations have been determined in full configuration spaces, leading to a significant difference from the previous results for some cases.
- Discrepancy between SM-WS and SM-HF still persists in the present calculations, in our opinion, it is due to the effects of Coulomb term used in each calculation.

Perspectives

- Correlations (Wigner, pairing, etc.) should be moved away from data before fits.
- Skyrme force should include ISB terms.

Summary

Thank you for your attention !