Mean-field Calculations of Hypernuclear Spectra

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Motivation

Why study hypernuclei?

- hypernucleus – system of protons, neutrons, and one or more hyperons
- hyperon serves as a deep probe in the nucleus
- study of hypernuclei helps with the understanding of nuclear structure and YN interactions
- hypothetically in dense nuclear matter – neutron stars

Production mechanisms of hypernuclei

- strangeness exchange reactions $K^+ + {}^A\mathrm{Z} \rightarrow {}^A_\Lambda\mathrm{Z} + \pi^+$
- associated production reactions $\pi^+ + {}^A\mathrm{Z} \rightarrow {}^A_\Lambda\mathrm{Z} + K^+$
- electroproduction of hypernuclei $e^- + {}^A\mathrm{Z} \rightarrow e^{-'} + K^+ + {}^A_\Lambda(\mathrm{Z}-1)$

Observed hypernuclei

- about 30 species of single-Λ hypernuclei from ${}^3_\Lambda\mathrm{H}$ to ${}^{208}_\Lambda\mathrm{Bi}$
- double-Λ hypernuclei: ${}^{6}_{\Lambda\Lambda}\mathrm{He}$, ${}^{10}_{\Lambda\Lambda}\mathrm{Be}$, and ${}^{13}_{\Lambda\Lambda}\mathrm{B}$
- other: only ${}^4_\Sigma\mathrm{He}$, no evidence of Ξ and Ω hypernuclei

Self-consistent mean-field model

mean-field model: protons, neutrons, and hyperons different particles placed in different potential wells – description of hypernuclei not limited by the number of particles in the system

spherical harmonic oscillator basis – $\mathrm{N}_{\mathrm{max}}$, $\hbar\omega$

Hamiltonian of a single-Λ hypernucleus

$$\widehat{H} = \widehat{T} + \widehat{V}^{\mathrm{NN}} + \widehat{V}^{\Lambda\mathrm{N}} - \widehat{T}_{\mathrm{CM}} \tag{1}$$

- nuclear core – Hartree-Fock method – nuclear mean field – realistic NN interaction $\mathrm{N}^2\mathrm{LO}_{\mathrm{opt}}$ [1] + density-dependent NN term [2] to mimic NNN interactions
  - Hartree-Fock method – mutual interactions between nucleons ⇒ mean field
- Λ interacts with the nuclear mean field through effective YNG ΛN interaction derived from Nijmegen model ESC08c [3]

[1] A. Ekström, et al., PRL **110**, 192502 (2013).
[2] H. Hergert et al., PRC **83**, 064317 (2011).
[3] M. Isaka et al., PRC **89**, 024310 (2014).

NN interactions

- N^2LO_{opt} [1] + density-dependent NN term [2]

DDNN term

$$\widehat{V}^{\mathrm{NN,DD}} = \frac{C_\rho}{6}(1+\widehat{P}_\sigma)\rho\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)\delta(\vec{r}_1-\vec{r}_2) \quad (2)$$

- contributes to the HF energy the same as contact three-body NNN interaction

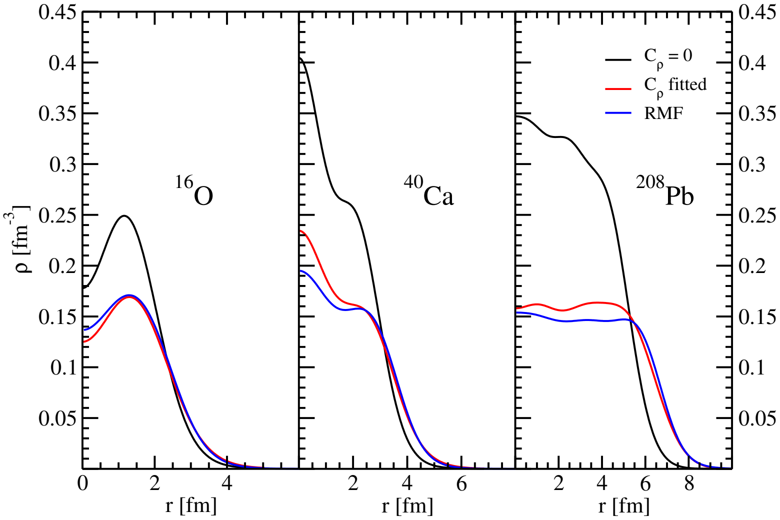
Contact NNN interaction

$$\widehat{V}^{\mathrm{NNN}} = C_{3\mathrm{N}}\delta(\vec{r}_1-\vec{r}_2)\delta(\vec{r}_2-\vec{r}_3) \quad (3)$$

[1] A. Ekström, et al., PRL **110**, 192502 (2013).

[2] H. Hergert et al., PRC **83**, 064317 (2011).

Nuclear density distributions



DDNN term is needed to obtain realistic density distributions

ΛN interaction

- YNG ΛN interaction derived from the Nijmegen model ESC08c [1]

Central part

$$G(r;k_{\mathrm{F}}) = \sum_{i=1}^{3} (a_i + b_i k_{\mathrm{F}} + c_i k_{\mathrm{F}}^2) \exp\left(-\frac{r^2}{\beta_i^2}\right) \tag{4}$$

a_i, b_i, c_i parameters which differ with spin and parity channels

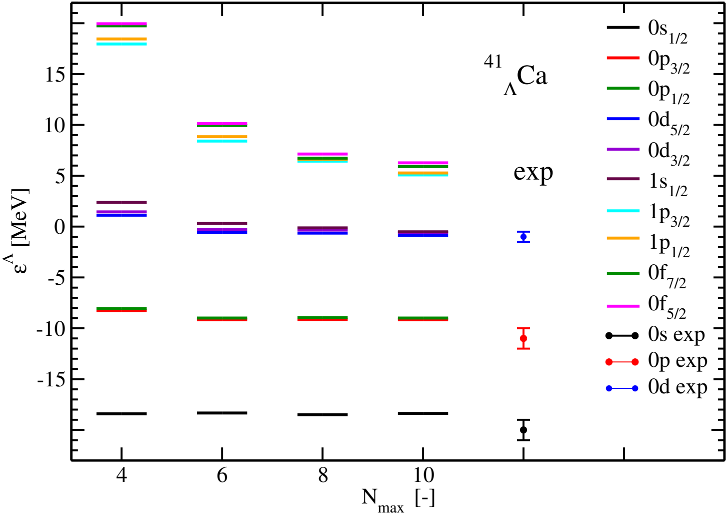
- SLS and ALS terms – Scheerbaum approximation

Fermi momentum k_{F} – Thomas-Fermi approximation

$$k_{\mathrm{F}} = \left(\frac{3\pi^2}{2}\langle\rho\rangle\right)^{1/3}, \quad \langle\rho\rangle = \int \mathrm{d}^3 r \; \rho_{\mathrm{N}}(\vec{r})\rho_{\Lambda}(\vec{r}) \tag{5}$$

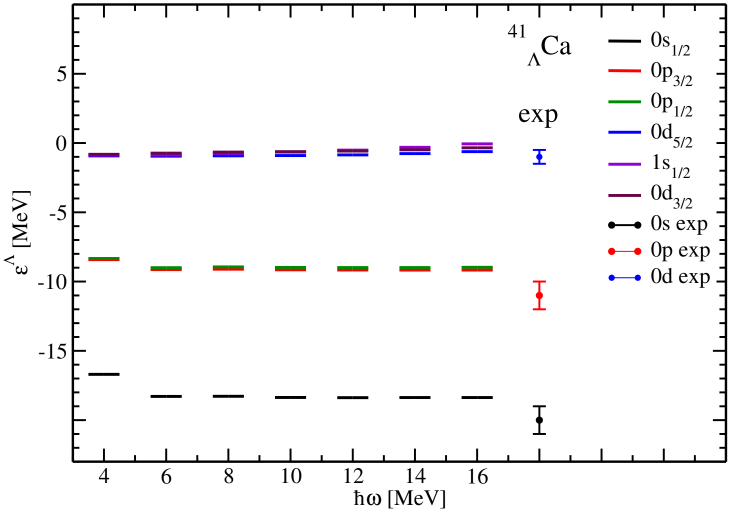
[1] M. Isaka et al., PRC **89**, 024310 (2014).

Convergence of the Λ single-particle spectra in $^{41}_{\Lambda}$Ca



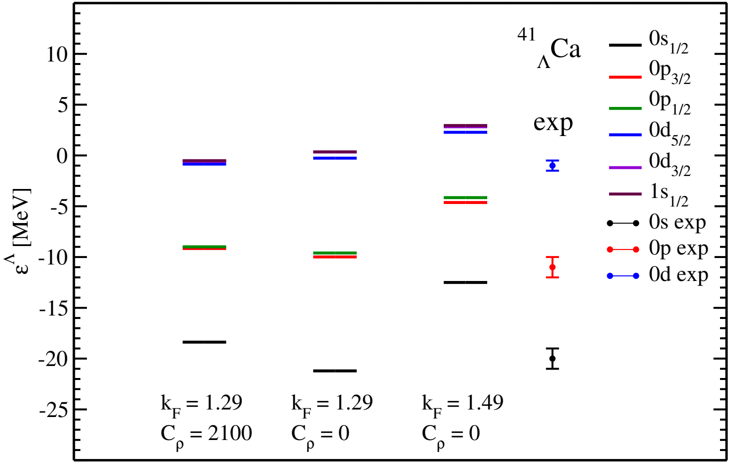
states with $\varepsilon^{\Lambda} > 0$ – possible excitations – we do not mention them further; data from BNL (π^+, K^+) [R. E. Chrien, Nucl. Phys. A **478**, 705c (1988)]

Stability of the Λ single-particle spectra in $^{41}_{\Lambda}$Ca



converged bound states stable; data from BNL (π^+, K^+) [R. E. Chrien, Nucl. Phys. A **478**, 705c (1988)]

Dependence of the Λ single-particle spectra in $^{41}_{\Lambda}$Ca on parameters C_ρ and k_F



C_ρ fitted, k_F fitted; k_F left from previous calculation; k_F fitted to $C_\rho = 0$; data from BNL (π^+, K^+) [R. E. Chrien, Nucl. Phys. A **478**, 705c (1988)]

Dependence of the Λ single-particle spectra in $^{17}_{\Lambda}$O on interactions

computing the Λ single-particle spectra in $^{17}_{\Lambda}$O with different NN and ΛN interactions

- new NN interaction CD-Bonn+$V_{\mathrm{low-k}}$ with cut-off parameter $\lambda = 2.6\ \mathrm{fm}^{-1}$ [1] + DDNN term [2]
- new YNG ΛN interaction derived from Nijmegen model ESC08a [3]

(i.) N^2LO_{opt} + YNG ESC08c

(ii.) N^2LO_{opt} + YNG ESC08a

(iii.) CD-Bonn + YNG ESC08c

(iv.) CD-Bonn + YNG ESC08a

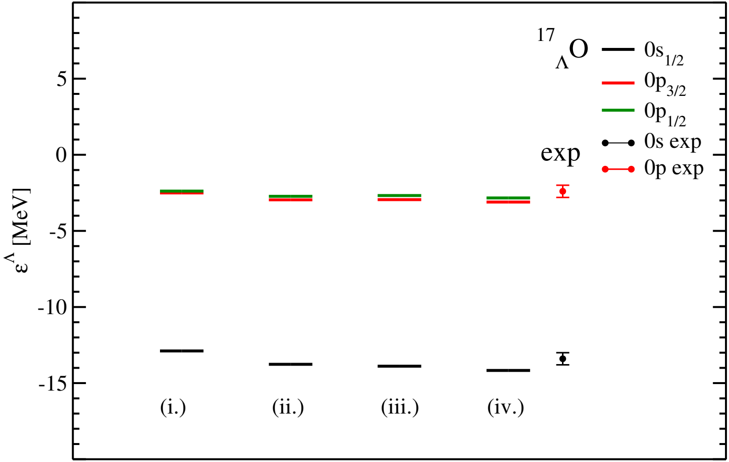
NN interaction CD-Bonn with λ (+ DDNN term) also gives realistic nuclear density distribution

[1] R. Machleidt, PRC **63**, 024001 (2001).

[2] H. Hergert et al., PRC **83**, 064317 (2011).

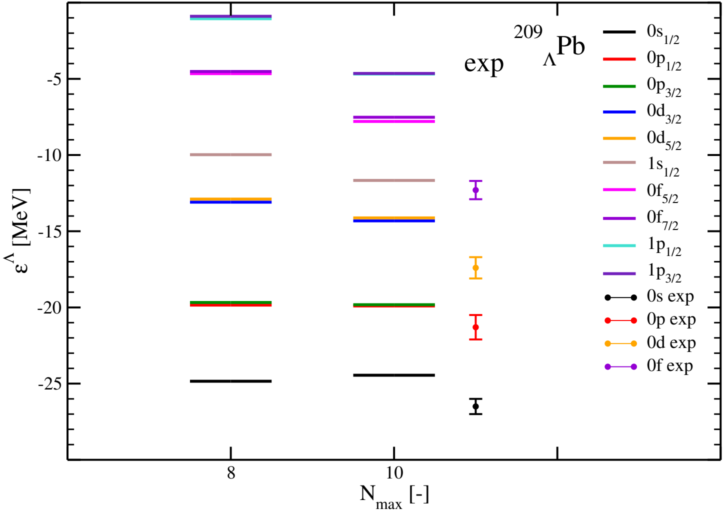
[3] Y. Yamamoto et al., Prog. Theor. Phys. Supp. **185**, 72 (2010).

Dependence of the Λ single-particle spectra in $^{17}_{\Lambda}$O on interactions



data from FINUDA $K^-_{stop} + {}^{A}\text{Z} \rightarrow {}^{A}_{\Lambda}\text{Z} + \pi^-$ [M. Agnello et al., Phys. Lett. B **698**, 219 (2011)]

Convergence of the Λ single-particle spectra in $^{209}_{\Lambda}$Pb



spectrum did not reach convergence, N_{max} is too small; data from KEK (π^+, K^+) [T. Hasegawa et al., Phys. Rev. C **53**, 1210 (1996).]

Conclusions and future plans

Conclusions

- we computed Λ single-particle energies in $^{41}_{\Lambda}$Ca, $^{17}_{\Lambda}$O, and $^{209}_{\Lambda}$Pb within the mean-field approach
- convergence and stability of the Λ bound states in $^{41}_{\Lambda}$Ca
- independence of the Λ spectrum in $^{17}_{\Lambda}$O on the choice of NN and ΛN interactions
- significant dependence of the Λ spectrum in $^{41}_{\Lambda}$Ca on C_ρ and k_{F}
- convergence of the Λ spectrum not reached in $^{209}_{\Lambda}$Pb $\Rightarrow$ the basis is too small, we would achieve satisfactory results with larger $\mathrm{N}_{\mathrm{max}}$

Future plans

- reduce computational complexity $\Rightarrow$ calculations performed in larger basis
- implement other realistic ΛN interactions $\Rightarrow$ EFT
- incorporate $\Lambda - \Sigma$ mixing and effect of the ΛNN interactions
- study core polarization effects and beyond mean-field calculations
- perform calculations in deformed single-particle basis