Quasi-dynamical Symmetries in the Backbending of Chromium Isotopes

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Overview

- Application: Back-bending in Band Structure of Chromium 48
- Method: Group Decomposition

  Advantages

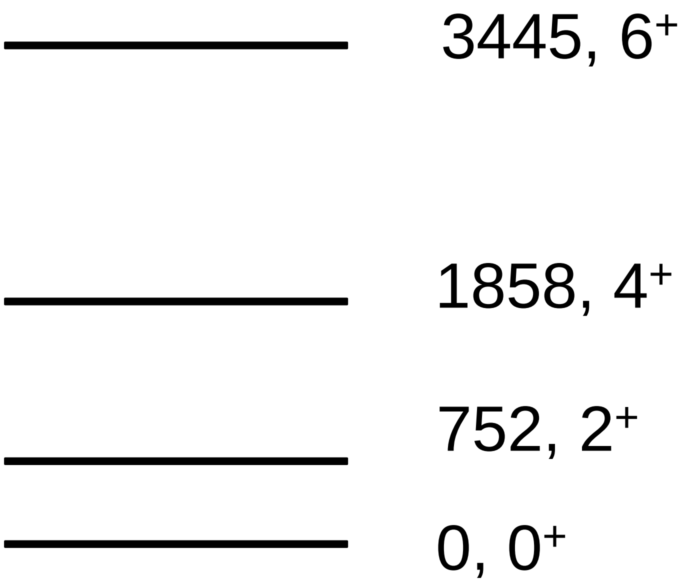
  ✓ New bases that can reduce matrix dimension

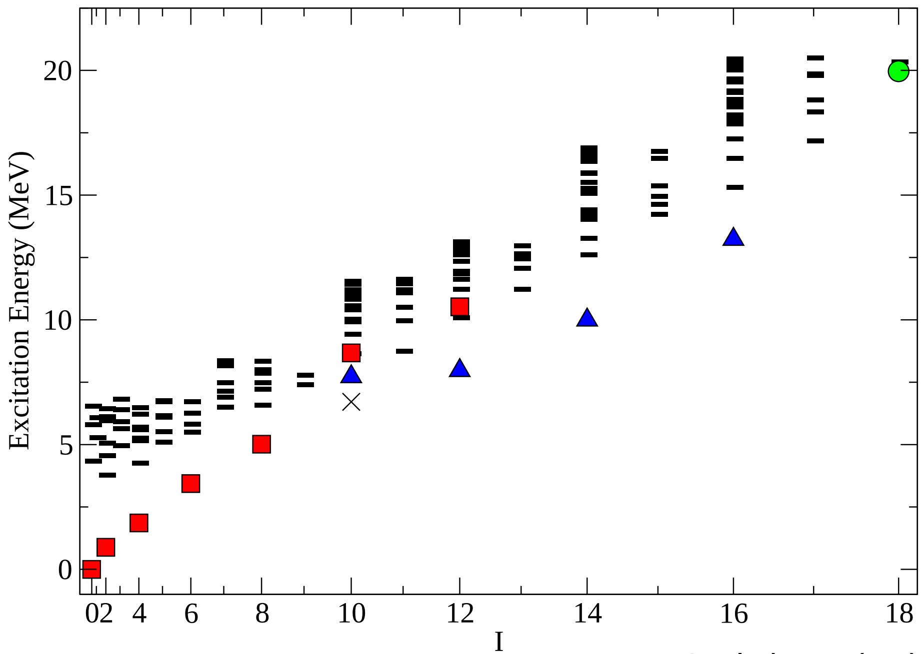
  ✓ Find most important parts of the wavefunction for truncation

  ✓ Another method to detect structures like rotational bands
- Results: Rotation Group – L and S separately, SU(3), SU(4)

Nuclear Spectra of ^{48}Cr

- On the right our calculated spectra, we identify two bands, red and blue, along the *yrast* states (lowest energy for each I). Linear implies rotational on this scale; do we have two rotational bands crossing? This has been explored before*.

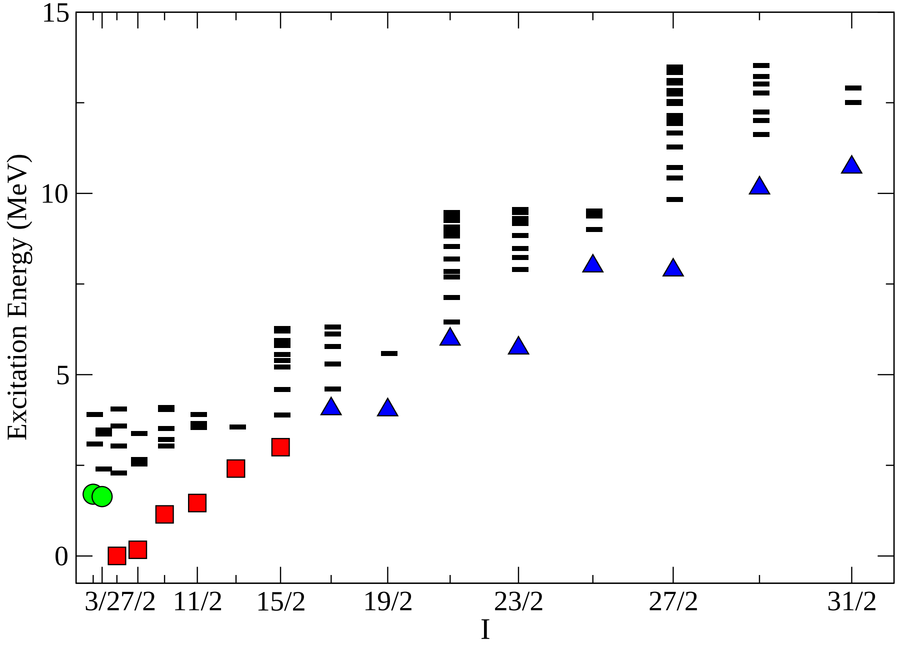


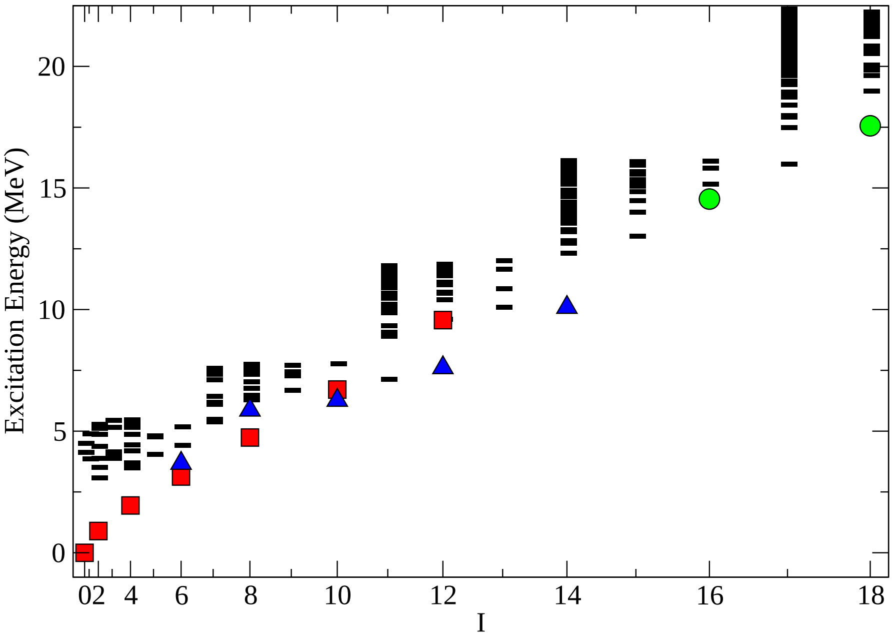


+Scale is as I(I+1)

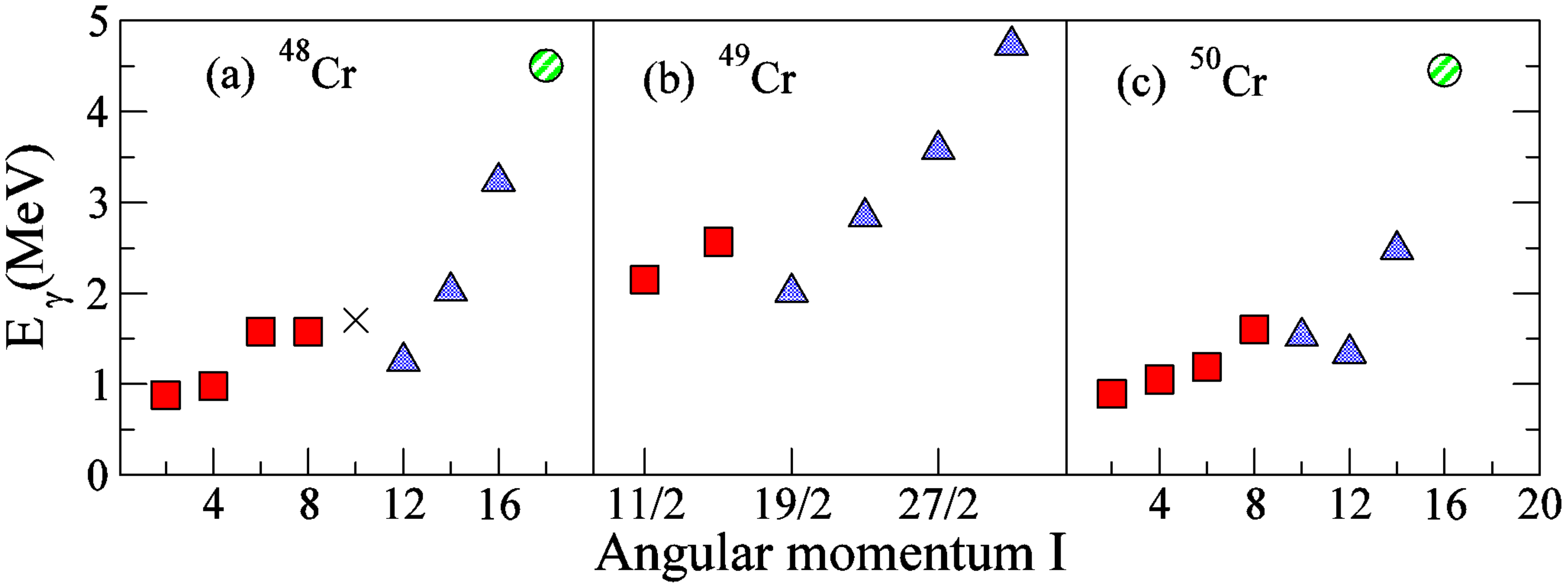
Experimental data (left): Cameron, Phys. Rev. C, 49 (1994), p. 1347
*For example Gao, Horoi, et. al. Phys. Rev. C 83, 057303 (2011)

Spectra of ^{49}Cr and ^{50}Cr





Backbending for Chromium Isotopes



$E_\gamma = E(I) – E(I\text{-}2)$

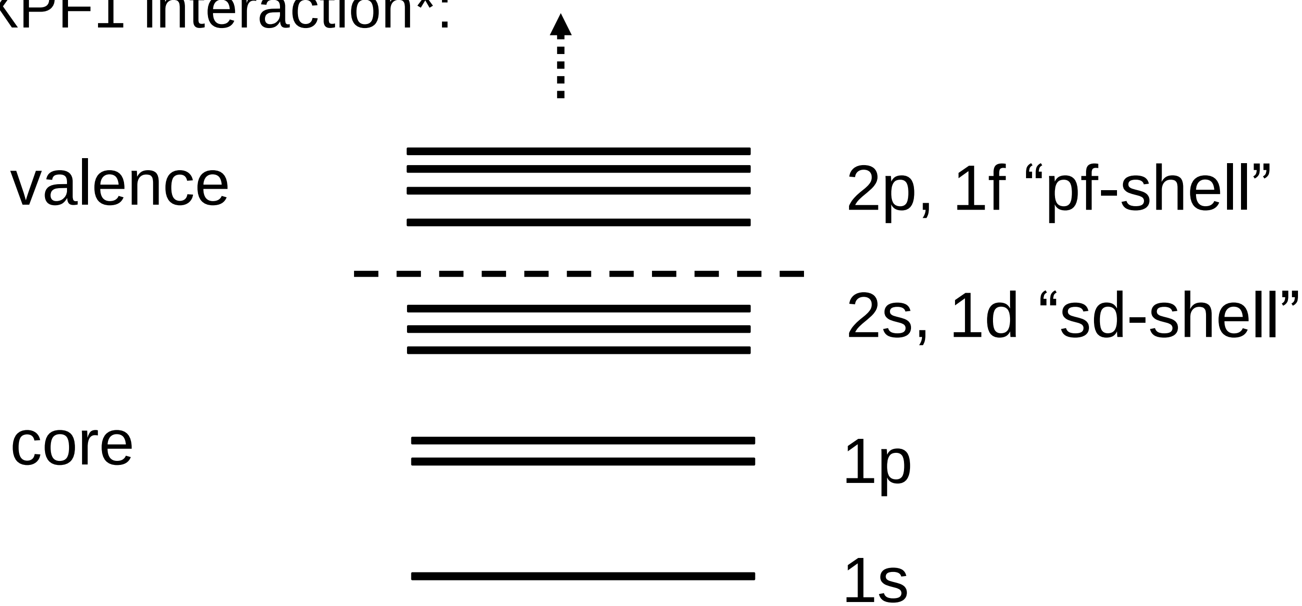
- Is backbending in these isotopes explained by band crossing?

The Nuclear Shell Model

- Hamiltonian

$$H = \sum_{i} \varepsilon_i + \frac{1}{2}\sum_{i \neq j} V_{ij}$$

- The shell model allows us a prescription to calculate nuclear structure. Chromium 48, 49, 50 lie in the pf shell just above the sd-shell which amounts to an inert Calcium 40 core. We used the GXPF1 interaction*:



Shell Model and Configuration Interaction

To solve we pick a basis, which are tensor products of single particle states. Often we truncate to a shell immediately after a closed shell nucleus, assuming the core to be inert. Using this we create a finite many-body basis, in occupation space, leading to the many body matrix Hamiltonian, H_{ij}. Any operator, e.g. a Casimir, can be obtatined as well.

$$\psi_i = \prod_{k=1}^{A} c_k^\dagger |0\rangle$$

$$\psi_i = |n\rangle \otimes |n\rangle \otimes \ldots \otimes |n\rangle$$

$$n = 0 \; or \; 1$$

$$\Psi = \sum_i c_i |\psi_i\rangle$$

$$H_{\mathrm{i}j} = \langle \Psi_i | H | \Psi_j \rangle$$

$$H_{\mathrm{i}j} c_j = E_n c_i$$

$$O_{\mathrm{i}j} = \langle \Psi_i | O | \Psi_j \rangle$$

$$O_{\mathrm{i}j} c_j = o_n \, c_i$$

Groups and Algebras

- Dynamical symmetries of a system are analyzed using Group Theory and Lie (Continuous) Algebras. A group, $\mathcal{G}$, in QM are basically sets of operators (e.g. matrices) that are closed under the group operation (e.g. matrix multiplication) and contain an identity operator.

$$A \cdot B = C, where\ A, B, C\ \in \mathcal{G}$$

- A Lie algebra, $\mathcal{A}$, is a vector space (include addition) of operators along with commutation, where certain members "generators" follows certain commutation relations.

$$[A, B] = C, where\ A, B, C\ \in\ \mathcal{A}$$

$$[J_i, J_j] = \epsilon_{ijk}\, J_K, for\ generators\ in\ algebra\ of\ rotation\ group\ SO(3)$$

Symmetries and Operators

- Important elements in a Lie Algebra are Casimirs, which commute with all other elements. For example, $\vec{J}^2$ a rotation group Casimir operator, and J_z, a group generator, where $\vec{J}$ is total angular momentum:

$$\vec{J} = \vec{L} + \vec{S} \text{ and } \left[\vec{J}^2, J_z\right] = 0$$

- $R(\alpha, \beta, \gamma)$ is a rotation a operator, a representation of an element in the rotation group, also denoted SO(3). Take around the z axis:

$$R(\alpha, \beta, \gamma) = e^{-iJ_z\alpha/\hbar} e^{-iJ_y\beta/\hbar} e^{-iJ_z\gamma/\hbar} \longrightarrow R(\phi) = e^{-iJ_z\phi/\hbar}$$

$$R(\phi)\psi(\alpha, \beta, \gamma) = \psi(\alpha + \phi, \beta, \gamma)$$

*Note: Notational difference here, but J=I in graphs pulled from our paper.

Symmetries and Conservation

- If there is a dynamical symmetry the Casimir will commute with the Hamiltonian, resulting in simultaneous eigenvectors.

$$\left[H, \vec{J}^{\,2}\right] = 0,\ [H, J_z] = 0 \text{ giving}$$

$$H\nu_j = \varepsilon_j \nu_j \quad \vec{J}^{\,2}\nu_j = j(j+1)\nu_j \quad J_z\nu_j = m\nu_j$$

- The operators will decompose into diagonal or block diagonal form with closed subspaces called *irreducible representations*, labelled by a quantum number (here j and size 2j+1). For example, the Wigner D-matrix, a representation of a rotation:

$$R(\alpha,\beta,\gamma)|jm\rangle = \sum_{mm\prime} \mathfrak{D}(R)^{(j)}_{mm\prime}|jm'\rangle$$

$$\mathfrak{D}^{j}_{mm\prime} = \begin{bmatrix} \mathfrak{D}^0_{00} & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & \mathfrak{D}^1_{-1-1} & \mathfrak{D}^1_{-1\,0} & \mathfrak{D}^1_{-1\,1} & 0 & 0 & \cdots \\ 0 & \mathfrak{D}^1_{0-1} & \mathfrak{D}^1_{0\,0} & \mathfrak{D}^1_{0\,1} & 0 & 0 & \cdots \\ 0 & \mathfrak{D}^1_{1-1} & \mathfrak{D}^1_{1\,0} & \mathfrak{D}^1_{1\,1} & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \mathfrak{D}^2_{-2-2} & \mathfrak{D}^2_{-2-1} & \cdots \\ 0 & 0 & 0 & 0 & \mathfrak{D}^2_{-1-2} & \mathfrak{D}^2_{-1-1} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Strength Functions

- The BIGSTICK code* has a built in function that uses Lanczos algorithm to quickly calculate operator matrix elements or strengths.

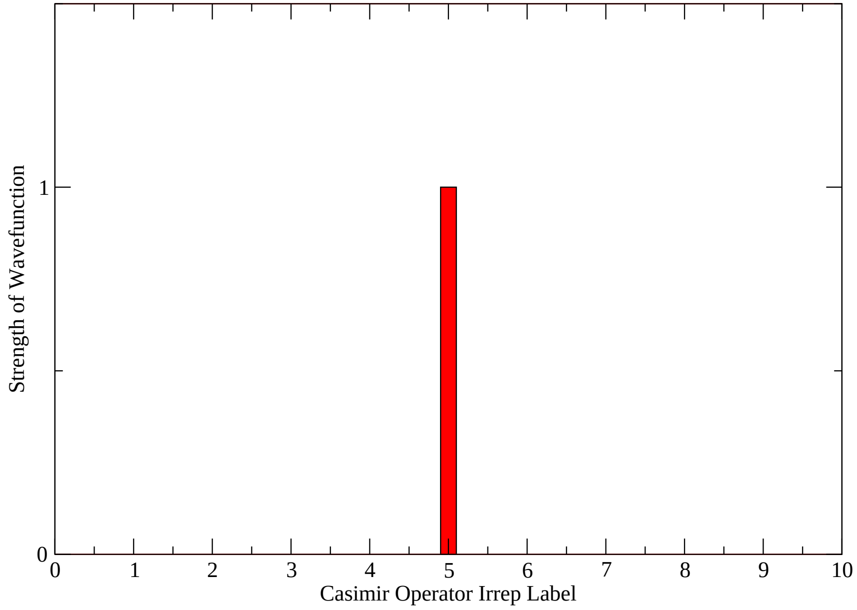
- We used it to decompose eigenstates of the nuclear Hamiltonian for a Casimir $\mathcal{C}$ into the irreducible representations, $\mathcal{C}_n$, of the group.

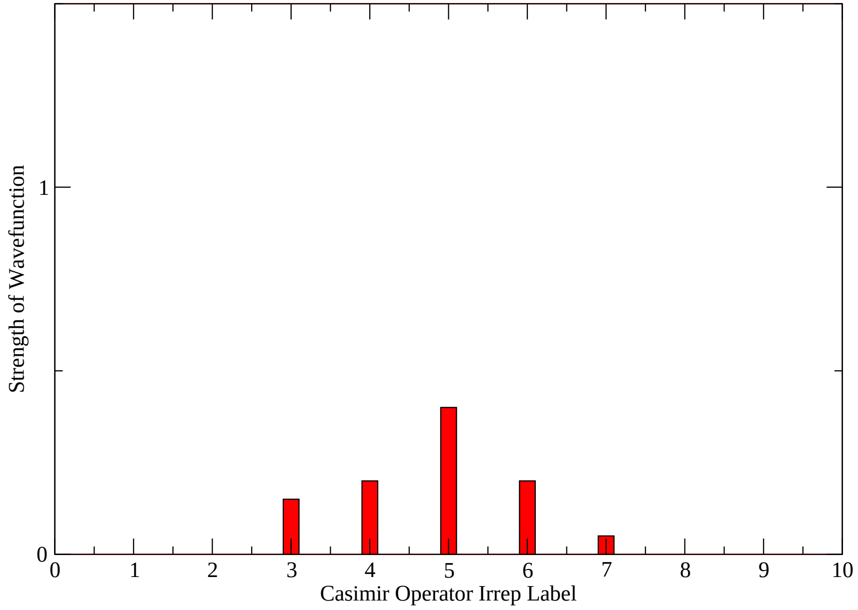
$$|\psi_1\rangle = \sum_n c(\mathcal{C}_n)\,|\mathcal{C}_n\rangle \qquad |\mathrm{c}(\mathcal{C}_n)|^2 = |\langle\mathcal{C}_n|\psi_1\rangle|^2$$

*Johnson et. al., Computer Physics Communications 184, p 2761 (2013)

Dynamical Symmetry

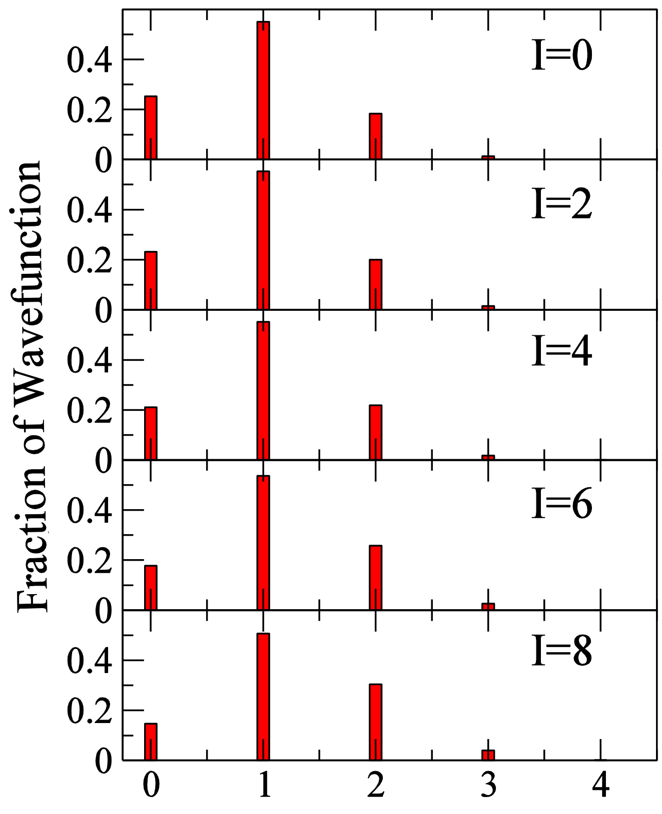
- If a dynamical symmetry we get an eigenstate of a Casimir, (left). Otherwise we get fragmentation (right).





Quasi-Dynamical Symmetry

- If the fragmented distributions persists (perhaps in a rotational band), this is a signature of *quasi-dynamical symmetry**



*D. J. Rowe, *Nucl. Phys. A 745 47*

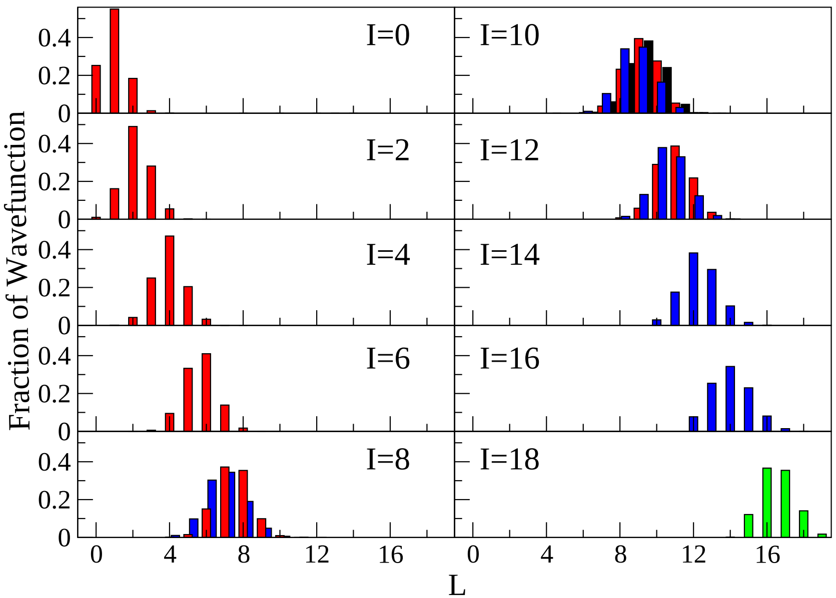
Symmetry Groups Analyzed

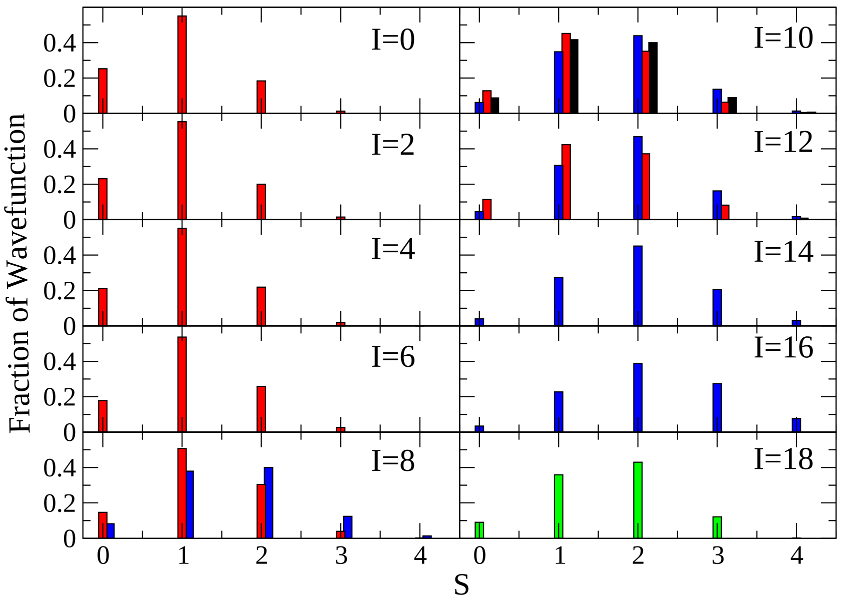
Casimir Operators (Some groups have multiple):

- Rotation Group for Orbital Motion
  - $\vec{L}^2$
- Rotation Group for Spin
  - $\vec{S}^2$
- Elliot SU(3) Fermion Collective Motion Model
  - $\frac{1}{4}(\vec{Q} \cdot \vec{Q} + 3\vec{L}^2)$
- SU(4) Wigner Spin-Isospin
  - $\vec{S}^2 + \vec{T}^2 + \vec{S}^2\vec{T}^2$
- $\mathcal{Q}$, T are Quadrupole and Iso-spin operators, respectively

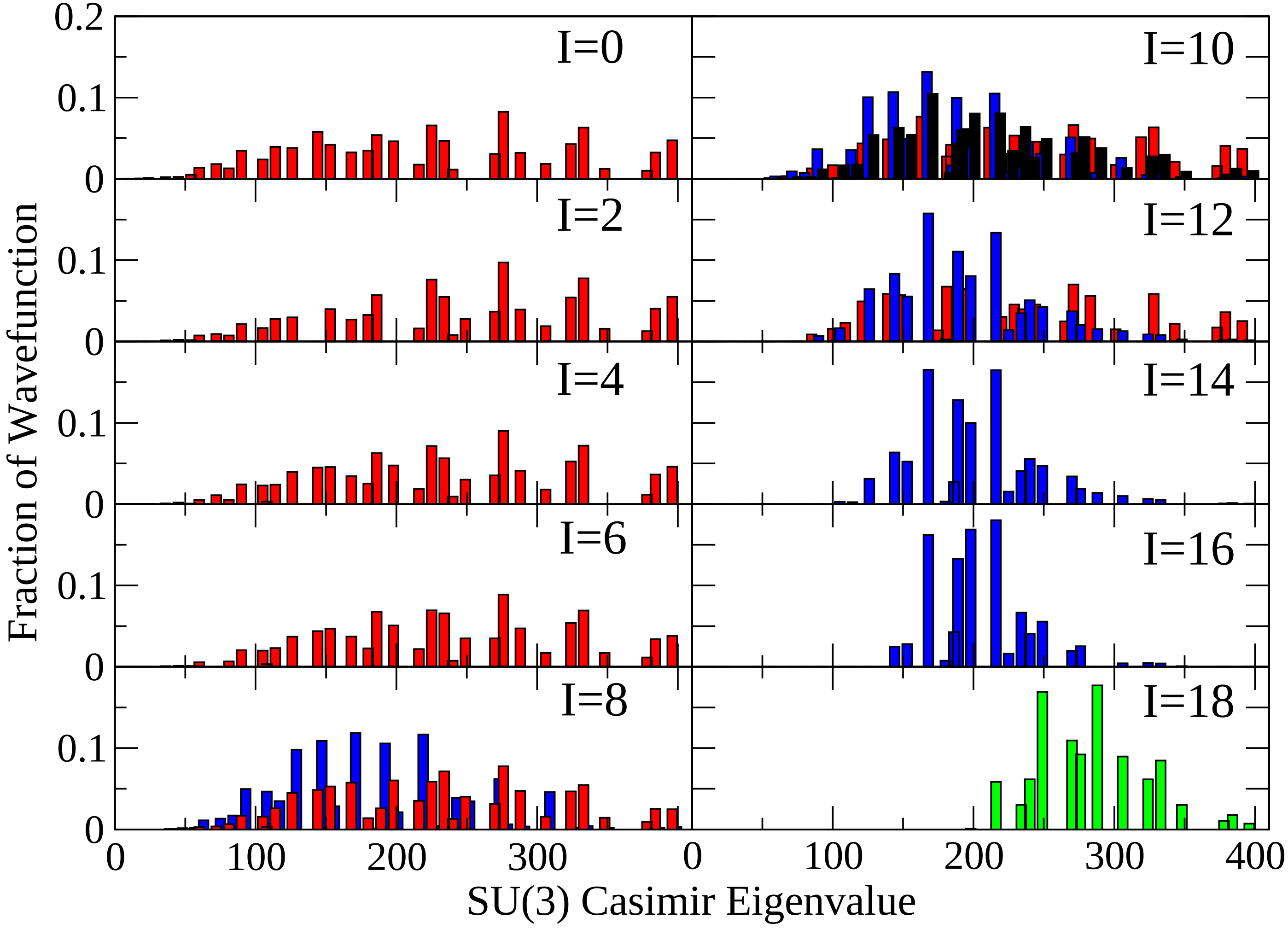
$$\mathcal{Q}_m = \sqrt{\frac{4\pi}{5}\left(\frac{r^2}{b^2} + b^2 p^2\right)} Y_{2m}(\theta, \phi)$$

Results for Chromium L and S of ^{48}Cr

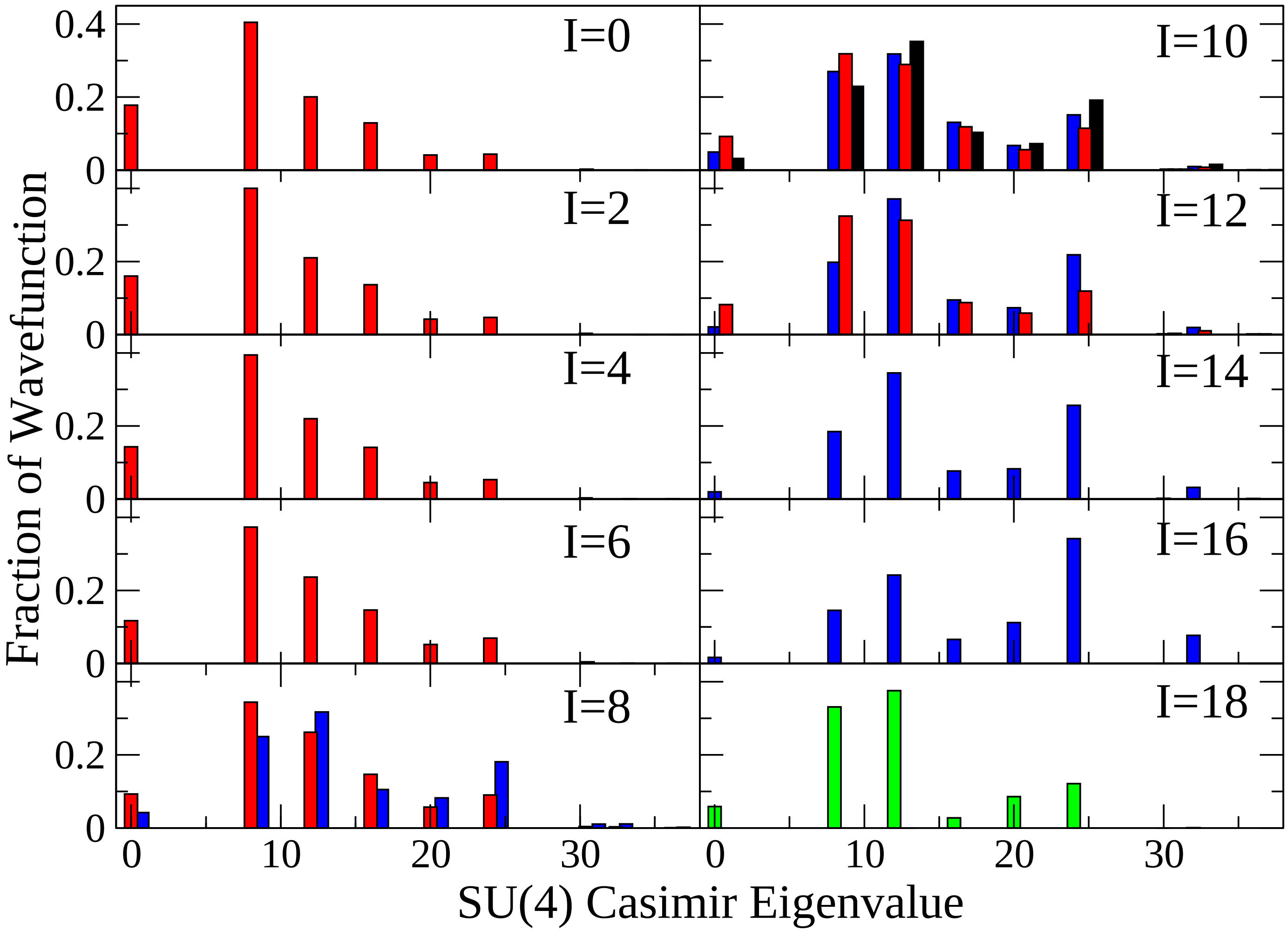




Results for Chromium SU(3) of ^{48}Cr



Results for Chromium SU(4) of ^{48}Cr



Results (Summary)

- In Chromium 48 we see two different bands crossing, similar results for 49, 50, could explain backbending
- Yrast state J=10 in Chromium 48 not in either band
- Both bands show quasi-dynamical symmetry
- Lower yrast band shows stronger coherence in SU(3) compared to the upper band

Current and Future Directions

- Analyzing symmetries of lighter nuclides and ab initio interactions
- Testing other symmetries, like isospin (proton versus neutron decompositions)

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- Organizers of ISS-28, thanks!

References:
- Herrera and Johnson, arXiv: 1607.00887 (this work)
- C. W. Johnson, Phys. Rev. C 91, 034313 (2015)