In-medium amplitudes and ‘minimal substitution’

comparing kaonic atoms with low energy pions

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Outline

- A short reminder: why $\sqrt{s}$ and how?
  $[s = (E_{K^-} + E_N)^2 - (\vec{p}_{K^-} + \vec{p}_N)^2]$.
- Pionic atoms: smooth variation of amplitudes with energy.
- $\pi^{\pm}$ scattering: sensitive tests of 'minimal subtitution' $(E \rightarrow E - V_c)$.
- Kaonic atoms: resonance structure of amplitudes.
- Results for kaonic atoms.
- Summary

WHY?

When describing meson-nucleus interaction by a $t\rho$ potential, with a free or effective t, the results could depend sensitively on $t(E,\rho)$.

Strong dependences on E near resonances require careful handling of these dependences.

Kaonic atoms: proximity of $\Lambda(1405)$.
PLB 702 (2011) 402, PRC 84 (2011) 045206, NPA 881 (2012) 150 and 159, NPA 899 (2013) 60.

Low energy pions: smooth dependence on E.
NPA 928 (2014) 128.

HOW?

Using the Mandelstam variable s in the nuclear medium
$s = (E_{K^-} + E_N)^2 - (\vec{p}_{K^-} + \vec{p}_N)^2$,

$$E_K = m_K - B_K, \qquad E_N = m_N - B_N.$$

In the nuclear medium $\vec{p}_{K^-} + \vec{p}_N \neq 0$.
Averaging over angles, $(\vec{p}_{K^-} + \vec{p}_N)^2 \to (p_{K^-})^2 + (p_N)^2$.
Substituting locally:

$$\frac{p_K^2}{2m_K} \to -B_K - \mathrm{Re}\; V_{\mathrm{opt}}^{K^-} - V_c,$$

$$\frac{p_N^2}{2m_N} \to T_N(\rho/\bar{\rho})^{2/3}$$

Defining $\delta\sqrt{s} = \sqrt{s} - E_{\rm th}$ with $E_{\rm th} = m_K + m_N$, then to first order in $B/E_{\rm th}$ and $(p/E_{\rm th})^2$ one gets

$$\delta\sqrt{s} = -B_N\rho/\bar{\rho} - \xi_N[T_N(\rho/\bar{\rho})^{2/3} + B_K\rho/\rho_0] + \xi_K[{\rm Re}\, V_{\rm opt} + V_c(\rho/\rho_0)^{1/3}],$$

with $\xi_N = m_N/(m_N + m_K)$, $\xi_K = m_K/(m_N + m_K)$, and $\bar{\rho}$ the average nuclear density.

The specific ρ/ρ_0 and $\rho/\bar{\rho}$ dependence ensures that $\delta\sqrt{s} \to 0$ when $\rho \to 0$.
E.F. and A. Gal, NPA 899 (2013) 60.

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Another variant is obtained when imposing minimal substitution (MS) requirement where E is replaced by $E - V_c$:

$$\delta\sqrt{s} = -B_N\rho/\bar{\rho} - \xi_N[T_N(\rho/\bar{\rho})^{2/3} + B_K\rho/\rho_0 + V_c(\rho/\rho_0)^{1/3}] + \xi_K{\rm Re}\ V_{\rm opt}.$$

Self-consistent solution required for Re $V_{\rm opt}$.
Identical numerical results obtained when the 1^{st} order expressions are replased by $\sqrt{s} - m_K - m_N$.

The Klein-Gordon equation is obtained by the 'Minimal Substitution' (MS) where $E \to E - V$, in analogy with the introduction of the Coulomb potential in a gauge invariant way.

Kolomeitsev, Kaiser and Weise, PRL 90 (2003) 092501: 'Importance of systematic incorporation of gauge invariance at all places where the pion energy appears explicitly, when solving the Klein-Gordon equation in the presence of electromagnetic interactions'.

Exotic atoms and nuclear densities

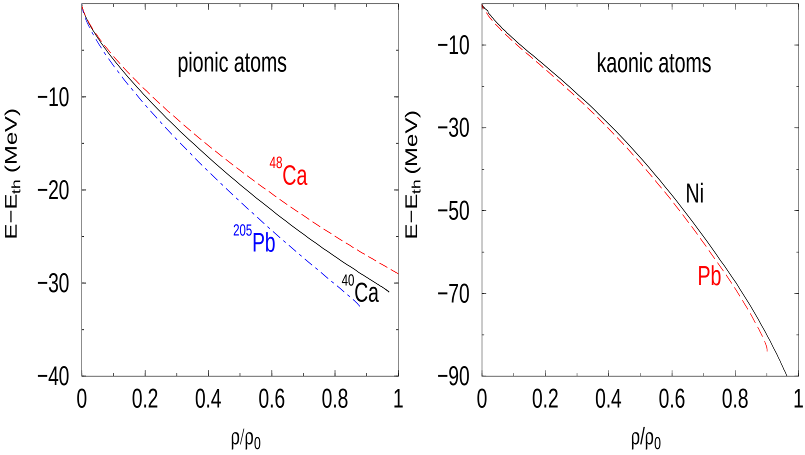
Proton densities ρ_p assumed known from nuclear charge.
Various *mean-field* calculations show linear dependence of differences between rms radii on $(N-Z)/A$:

$$r_n - r_p = \gamma \frac{N-Z}{A} + \delta \ .$$

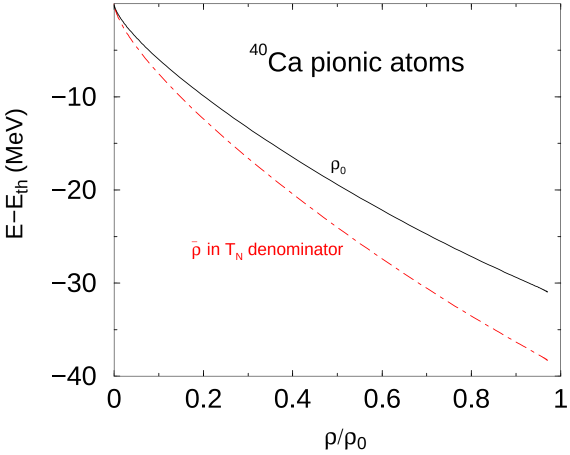
Use 2pF densities

$$\rho_{n,p}(r) = \frac{\rho_{0n,0p}}{1+\exp((r-R_{n,p})/a_{n,p})} \ ,$$

different shapes for ρ_n are 'skin' $(a_n = a_p)$,
'halo' $(R_n = R_p)$ and their average.
Fits to exotic atoms data *vs* neutron radius parameter γ.



Examples of the density-to-energy transformation, without MS.



Sensitivity to assumptions and parameters

Low energy pions: the power of experimental constraints.

The Ericson-Ericson extension of the Kisslinger potential.

$$2\mu V_{\mathrm{opt}}(r) = q(r) + \vec{\nabla}\cdot\alpha(r)\vec{\nabla}$$

$$\begin{aligned} q(r) \;&=\; -4\pi(1+\frac{\mu}{M})\{b_0[\rho_n(r)+\rho_p(r)] + b_1[\rho_n(r)-\rho_p(r)]\} \\ &\quad -4\pi(1+\frac{\mu}{2M})4B_0\rho_n(r)\rho_p(r), \end{aligned}$$

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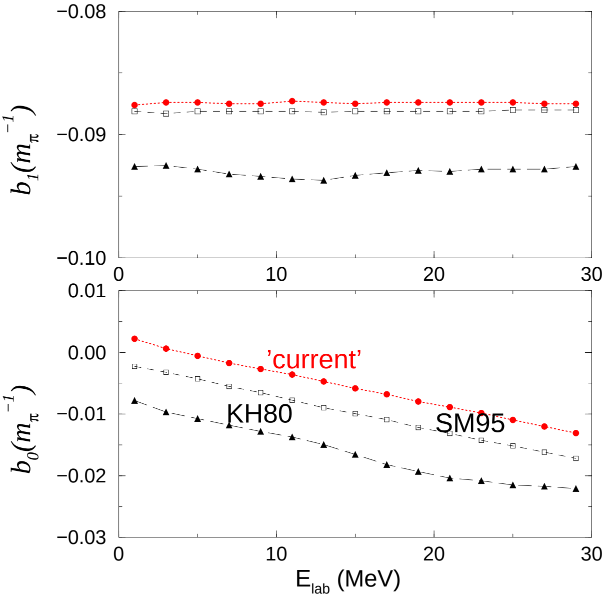
$$\begin{aligned} \alpha(r) &= 4\pi(1+\frac{\mu}{M})^{-1}\{c_0[\rho_n(r)+\rho_p(r)] + c_1[\rho_n(r)-\rho_p(r)]\} \\ &\quad +4\pi(1+\frac{\mu}{2M})^{-1}4C_0\rho_n(r)\rho_p(r). \end{aligned}$$

Dependence on α is confined to the surface.
Medium modifications: study $q(r)$, mostly b_1. ($b_0 \approx 0$).
Experimental: 100-120 good quality data points for pionic atoms.



Empirical pion-nucleon *s*-wave amplitudes

Amplitudes in the medium:

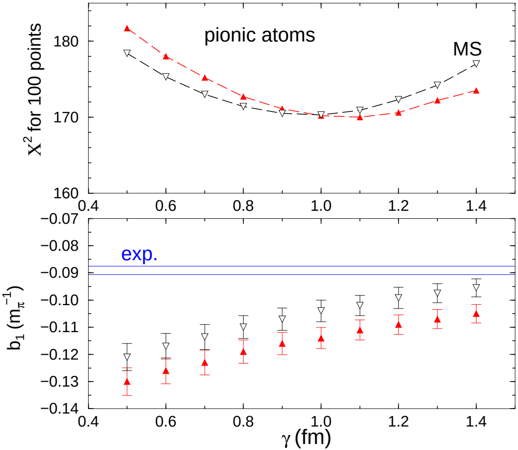
Double-scattering contributions for Pauli correlated nucleons

$$\overline{b}_0 = b_0 - \frac{3}{2\pi}(b_0^2 + 2b_1^2)p_F,$$

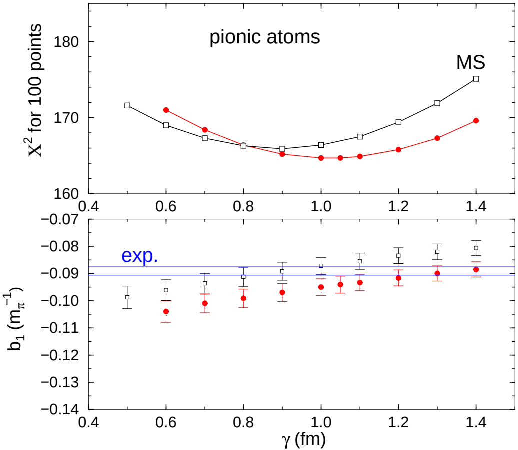
where p_F is the local Fermi momentum corresponding to the local nuclear density $\rho = 2p_F^3/(3\pi^2)$.

$$\alpha(r) \rightarrow \frac{\alpha(r)}{1+\frac{1}{3}\xi\alpha(r)}$$

($\xi \approx 1$ EELL correction.)



Using empirical $b_0(E)$, b_1 independent of ρ.

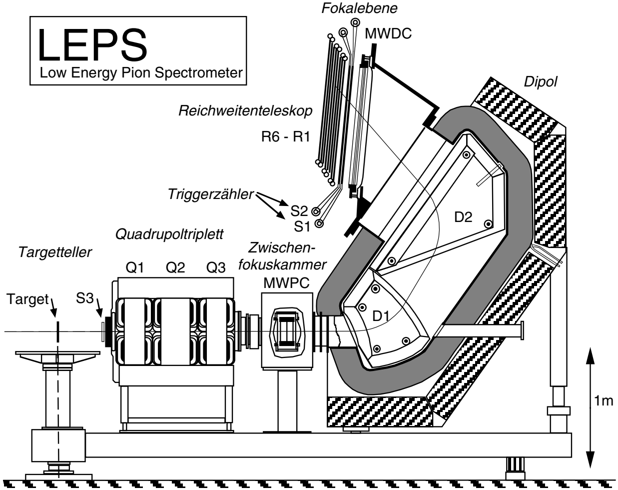


Using empirical $b_0(E)$ and Weise's ansatz

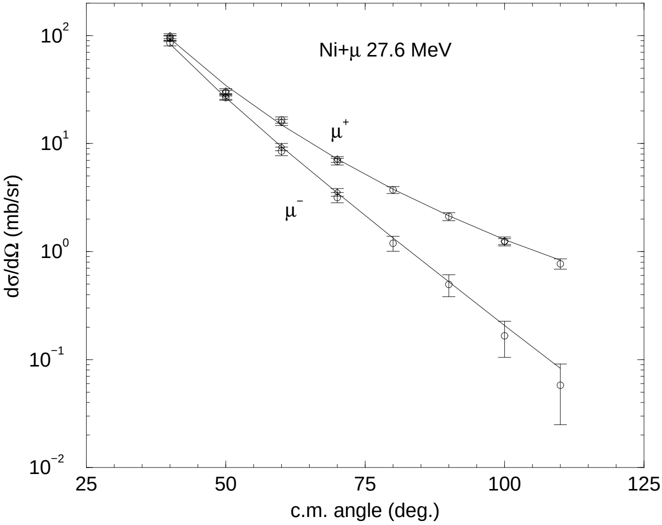
$$b_1(\rho) = \frac{b_1}{1 - \sigma\rho/m_\pi^2 f_\pi^2} = \frac{b_1}{1 - 2.3\rho}, \quad \rho \text{ in fm}^{-3}.$$

Tests with low-energy pion scattering

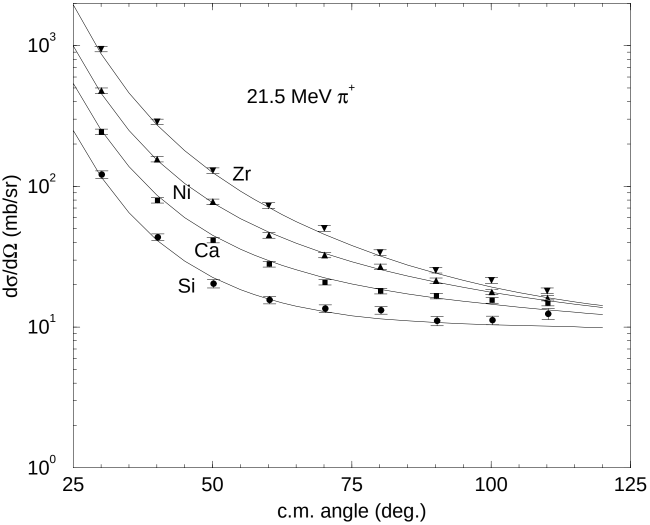
Precision measurements of elastic scattering of 21.5 MeV π^+ AND π^- by several nuclei were performed at PSI with parallel measurements of Coulomb scattering of muons for normalization.



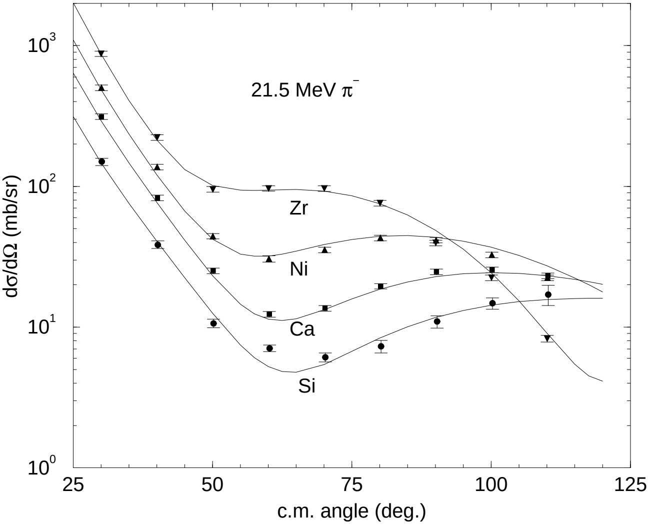
Pion beams accompanied by muon beams. Both scattered and detected in parallel. Minimise errors on π^+ to π^- differences.



Absolute calibration with muons.



Experiment and fits for π^+.



Experiment and fits for π^-.

Testing in-medium algorithms with pions

Pionic atoms (χ^2/df=1.74)

$\delta\sqrt{s}$	no	yes	yes
MS($E - V_c$)	no	no	yes
χ^2 for 100 points	176	164	165

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$\pi^\pm$ scattering (21.5 MeV, χ^2/df=1.25)

$\delta\sqrt{s}$	no	yes	yes
MS($E - V_c$)	yes	yes	yes
V_c in Im C_0^*	yes	no	yes
χ^2 for 72 points	92	88	83

* Im $C_0 = C_0^0 + \beta V_c$. Good consistency between 21.5 MeV and atoms. Disagreement with free b_1 if 'no MS'.

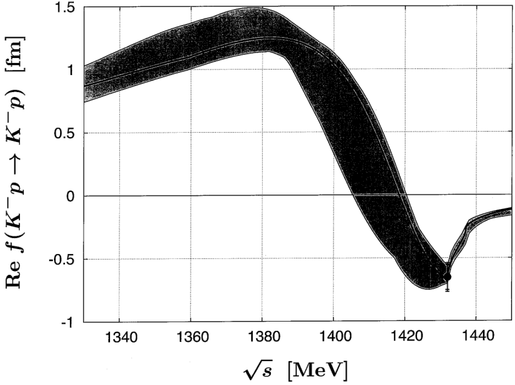
Pionic atoms: very good fits, significantly reduced χ^2 when using the $\delta\sqrt{s}$ with empirical E-dependence.

Pion scattering ($\pm$): very good fits, significantly reduced χ^2 when using the $\delta\sqrt{s}$ with empirical E-dependence.
Significantly reduced χ^2 when applying $E \to E - V_c$ (MS).
Good consistency between 21.5 MeV and atoms.

Introducing $\delta\sqrt{s}$ allows differences between different elements.
More so when MS is also included.

Best-fit values of b_1 are insensitive to variations in models, both for atoms and for scattering fits to experiment.
Best-fit b_1 supports the Weise's ansatz.

Kaonic atoms: resonance structures



From Y. Ikeda, T. Hyodo, W. Weise, NPA 881 (2012) 98.

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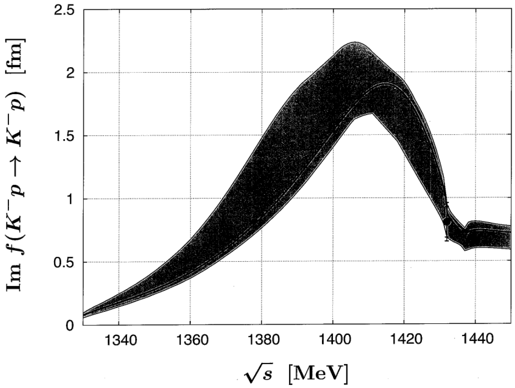

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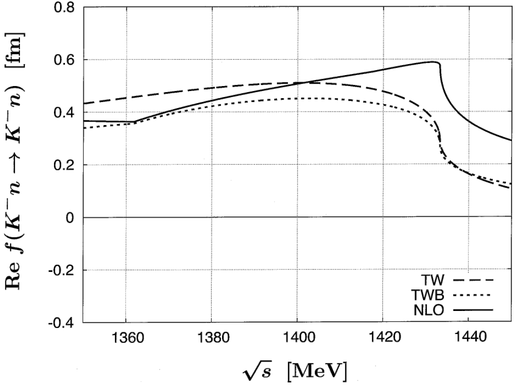

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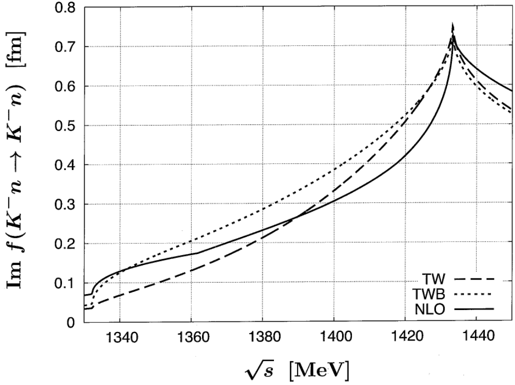

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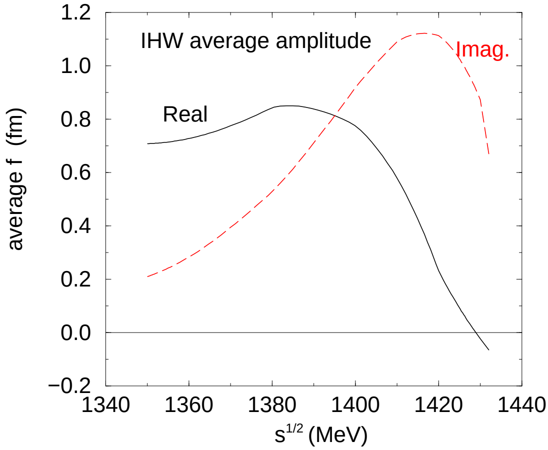

K^- amplitudes in the medium:

$$2\mu_K V^{(1)}_{K^-}(\rho) = -4\pi \left[\frac{(2\tilde{f}_{K^-p} - \tilde{f}_{K^-n})\,\frac{1}{2}\rho_p}{1 + \frac{1}{4}\xi_{k=0}\tilde{f}_0\rho(r)} + \frac{\tilde{f}_{K^-n}(\frac{1}{2}\rho_p + \rho_n)}{1 + \frac{1}{4}\xi_{k=0}\tilde{f}_1\rho(r)} \right] .$$

For kaonic atoms $k \approx 0$ and then $\xi_{k=0} = 9\pi/p_F^2$, with p_F the local Fermi momentum.

E.F and A. Gal, NPA 899 (2013) 60.

Kaonic atoms: adding a phenomenological multi-nucleon term:

$$[b + B(\frac{\rho}{\rho_0})^\alpha]\rho$$

$\delta\sqrt{s}$	MS	Re b	Re B	Im B	α	$\chi^2(65)$
0.	no	-	-	-	-	747
0.	no	-0.51±0.07	1.48±0.07	-0.41±0.06	4.2±0.4	223

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yes	no	-	-	-	-	2115

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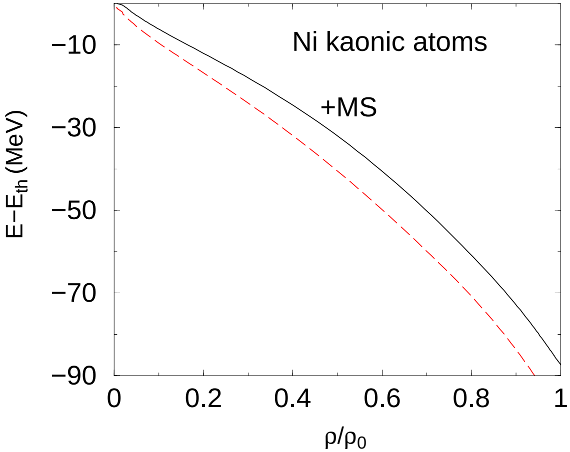
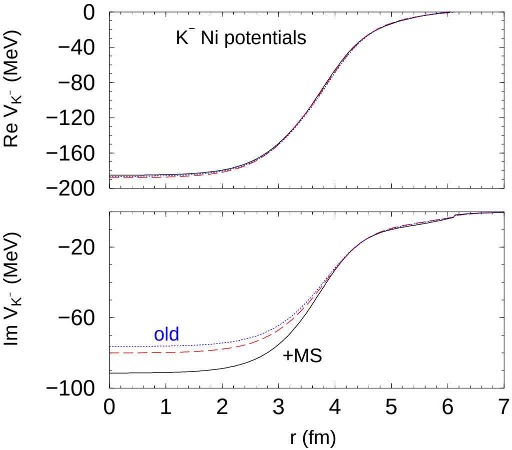
yes	no	-	-	-	-	2115
yes	no	-0.36$\pm$0.08	1.90$\pm$0.21	0.83$\pm$0.19	1.07$\pm$0.17	117
yes	yes	-0.23$\pm$0.06	1.80$\pm$0.21	0.95$\pm$0.19	1.44$\pm$0.21	108

$\chi^2/df = 1.8$

Again MS ($E \to E - V_c$) introduces additional flexibility without adding parameters.


Ni kaonic atoms
+MS
−10
−30
−50
−70
−90
0
0.2
0.4
0.6
0.8
1
E−E$_{th}$ (MeV)
ρ/ρ$_0$

The role of experimental constraints



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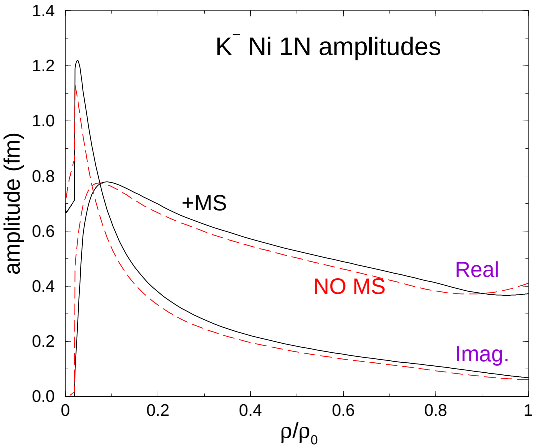
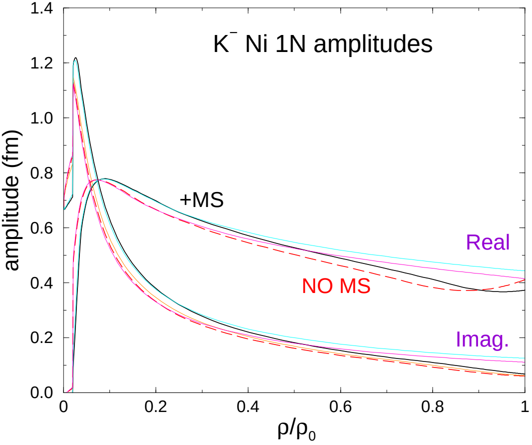


The role of experimental constraints



Summary

- Exotic-atom potentials constructed from meson-nucleon amplitudes near threshold and tested on pionic atoms, $\pi^{\pm}$ scattering and kaonic atoms.
- In-medium $\delta\sqrt{s}$ improve fits to data without additional parameters.
- Minimal substitution $E \to E - V_C$ further improves fits.
- Previous conclusions confirmed, including support for medium-modification of f_π and empirical multi-nucleon terms for kaonic atoms.

Acknowledgemdnts

Countless discussions with Avraham Gal throughout this work are gratefully acknowledged.

The sub-threshold approach was applied in the last three years to several problems together with A. Cieplý, A. Gal, D. Gazda, and J. Mareš.

Tomozawa-Weinberg (TW) lowest-order chiral limit is:
$b_0 = 0, \qquad b_1 = -\frac{\mu_{\pi N}}{8\pi f_\pi^2} = -0.079\ m_\pi^{-1}$.
Double-scattering contributions lead to $\overline{b}_0 = b_0 - \frac{3}{2\pi}(b_0^2 + 2b_1^2)p_F$, where p_F is the local Fermi momentum.
In-medium renormalization of the pion decay constant f_π, given to first order in the nuclear density ρ by $\frac{f_\pi^2(\rho)}{f_\pi^2} = \frac{<\bar{q}q>_\rho}{<\bar{q}q>_0} \simeq 1 - \frac{\sigma\rho}{m_\pi^2 f_\pi^2}$, where $< \bar{q}q >_\rho$ stands for the in-medium chiral condensate and $\sigma \simeq 50$ MeV is the pion-nucleon σ term.
This leads to the Weise's ansatz:

$$b_1(\rho) = \frac{b_1}{1 - \sigma\rho/m_\pi^2 f_\pi^2} = \frac{b_1}{1 - 2.3\rho}.$$

when ρ is in fm^{-3}.