${}^{3}_{\Lambda}$n & other neutron-rich hypernuclei

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- 1st sound calculation showing that ${}^{3}_{\Lambda}$n is **unbound** is due to Downs-Dalitz, PR 114 (1959) 593. However, the HypHI Collaboration, Rappold et al. PRC 88 (2013) 041001(R), argued recently by observing $\pi^{-}+{}^{3}$H weak decay that ${}^{3}_{\Lambda}$n is **bound**.
- **Recent calculations agree on unbound ${}^{3}_{\Lambda}$n:**
  (i) Garcilazo-Valcarce, PRC 89 (2014) 057001
  (ii) Hiyama-Ohnishi-Gibson-Rijken, *ibid* 061302(R)
  (iii) Gal-Garcilazo, PLB 736, 93-97.
- **We derived constraints from several hypernuclear systems to rule out a bound ${}^{3}_{\Lambda}$n.**

- Λ hyperon stabilizes nuclear cores, acting as a glue [Dalitz & Levi Setti, Nuovo Cimento 30 (1963) 489] ${}^{6}_{\Lambda}$He, ${}^{7}_{\Lambda}$Be, ${}^{8}_{\Lambda}$He, ${}^{9}_{\Lambda}$Be, ${}^{10}_{\Lambda}$B observed in emulsion.
- The lightest unstable-core hypernucleus ${}^{6}_{\Lambda}$H was predicted by DLS, reinforced in estimates by Majling [NPA 585 (1995) 211c] with $B_{\Lambda}^{\rm Dalitz}({}^{6}_{\Lambda}{\rm H}) = 4.2$ MeV. Akaishi (1999) predicted $B_{\Lambda}^{\rm Akaishi}({}^{6}_{\Lambda}{\rm H}) = 5.8$ MeV.
- FINUDA found three ${}^{6}_{\Lambda}$H events in ${}^{6}{\rm Li}(K^{-}_{\rm stop}, \pi^{+})$ Production rate: $R(\pi^{+}) = (5.9 \pm 4.0) \cdot 10^{-6}/K^{-}_{\rm stop}$ [PRL 108 (2012) 042501, NPA 881 (2012) 269]. $B_{\Lambda}({}^{6}_{\Lambda}{\rm H}) = (4.0 \pm 1.1)$ MeV *vs* (3.9 ± 0.1) MeV calc. by Gal-Millener, PLB 725 (2013) 445.
- E. Hiyama et al. NPA 908 (2013) 29: **unbound**.

Simple considerations

s-shell Λ hypernuclei

Hypernucleus	J^π(g.s.)	B_Λ MeV	J^π	E_x MeV
${}^{3}_{\Lambda}$H	$1/2^+$	0.13(5)		
${}^{4}_{\Lambda}$H	0^+	2.04(4)	1^+	1.04(5)
${}^{4}_{\Lambda}$He	0^+	2.39(3)	1^+	1.15(4)
${}^{5}_{\Lambda}$He	$1/2^+$	3.12(2)		

Past "Exact" Calculations

- $A = 3$ K. Miyagawa et al., PRC 51 (1995) 2905 Faddeev.
- $A = 3, 4$ A. Nogga et al., PRL 88 (2002) 172501
  Faddeev and Faddeev-Yakubovsky.
- $A = 4$ E. Hiyama et al., PRC 65 (2002) 011301(R)
  Jacobi-coordinate Gaussian basis.
- $A = 3, 4, 5$ H. Nemura et al., PRL 89 (2002) 142504
  Stochastic variation with correlated Gaussians.

Incompatibility of bound-state ${}^{3}_{\Lambda}$n with Λp scattering

	$B_\Lambda^{T=0}$=0		$B_\Lambda^{T=0}$=0.13 MeV		$B_\Lambda^{T=1}$=0 (${}^{3}_{\Lambda}$n just bound)		
$r_{\rm eff}$ (fm)	a (fm)	$\sigma^{\rm tot}_{\Lambda p}$ (mb)	a (fm)	$\sigma^{\rm tot}_{\Lambda p}$ (mb)	a (fm)	$\sigma^{\rm tot}_{\Lambda p}$ (mb)	$B_\Lambda^{T=0}$ (MeV)
2.5	−1.185	129.7	−1.498	192.5	−4.491	953.8	2.59
3.5	−1.405	152.4	−1.895	239.7	−5.930	943.1	1.74

- Solve ΛNN Faddeev equations. Separable NN, ΛN interactions fitted to low-energy scattering, neglecting ΛN spin dependence for simplicity.
- **If ${}^{3}_{\Lambda}$n is bound, ${}^{3}_{\Lambda}$H is substantially overbound. Only $B_\Lambda^{T=0}$=0.13±0.05 MeV is consistent with $\sigma^{\rm tot}_{\Lambda p}(p_\Lambda$=145±25 MeV/c)=180±22 mb.**

- In next step relax ΛN spin independence: $a_s = -2.03$, $r_s = 3.66$, $a_t = -1.39$, $r_t = 3.32$ fm, and scale up $V_t(\Lambda N) \to x V_t(\Lambda N)$, $x \geq 1$, until ${}^{3}_{\Lambda}$n binds (Fredholm determinant vanishes at $E = 0$).
- This allows us to also study ${}^{3}_{\Lambda}\mathrm{H}(\frac{3}{2}^{+})_{\text{exc.}}$ vs. ${}^{3}_{\Lambda}\mathrm{H}(\frac{1}{2}^{+})_{\text{g.s.}}$.

x	${}^{3}_{\Lambda}$n FD$(E = 0)$	$B_{\Lambda}[{}^{3}_{\Lambda}\mathrm{H}^{T=0}(\frac{1}{2}^{+})]$	$B_{\Lambda}[{}^{3}_{\Lambda}\mathrm{H}^{T=0}(\frac{3}{2}^{+})]$
1.00	0.55	0.096	unbound
1.10	0.47	0.147	0.124
1.20	0.39	0.211	0.448
...	...	...	...
1.61	+0.004	0.625	3.772
1.62	−0.006	0.638	3.890

- **${}^{3}_{\Lambda}\mathrm{H}(\frac{3}{2}^{+})$ becomes g.s. well before ${}^{3}_{\Lambda}$n binds.**

$\Lambda - \Sigma$ coupling considerations

$\Lambda - \Sigma$ coupling for ${}^4_\Lambda$H and ${}^4_\Lambda$He

Y. Akaishi et al., PRL 84 (2000) 3539

$$|{}^4_\Lambda\mathrm{He}(\mathrm{T}=1/2)\rangle = \alpha s^3 s_\Lambda + \beta s^3 s_\Sigma$$

From ΛN−ΣN g matrix for 0s orbits

$v = \langle s^3 s_\Lambda | g | s^3 s_\Sigma \rangle$, $\quad \Delta E \sim 80$ MeV $\quad {}^3g_{ss} = 4.8 \quad {}^1g_{ss} = -1.0$

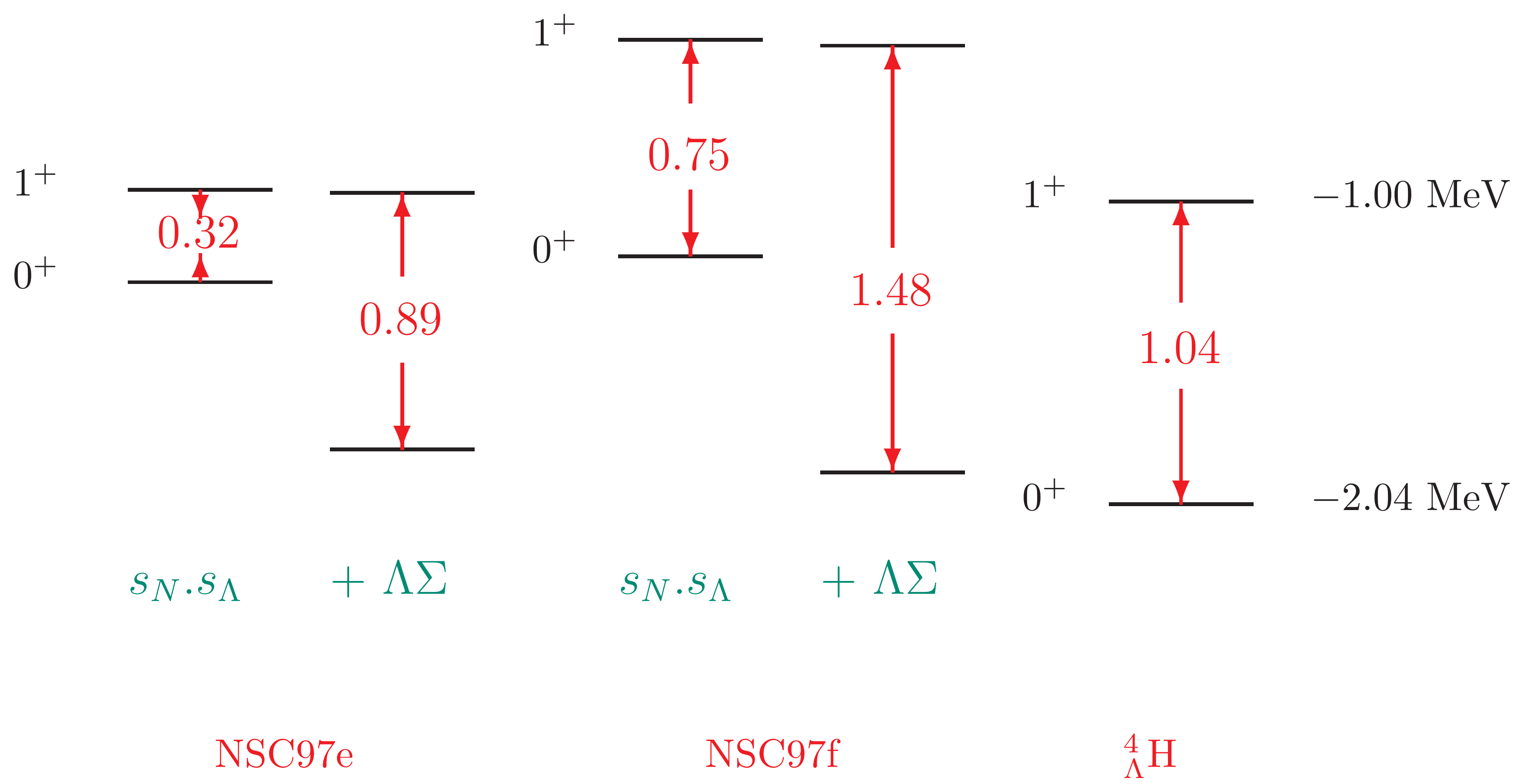
0^+	$v = \frac{3}{2}\,{}^3g_{ss} - \frac{1}{2}\,{}^1g_{ss}$	Admixture $\sim -v/\Delta E$
1^+	$v = \frac{1}{2}\,{}^3g_{ss} + \frac{1}{2}\,{}^1g_{ss}$	$E^{\mathrm{shift}} \sim v^2/\Delta E$

NSC97f: for 0^+ $\quad v \sim 7.6$ MeV $\quad \Rightarrow \quad E^{\mathrm{shift}} \sim 0.72$ MeV

comparable to genuine ΛN splitting $\Delta_s = 0.75$ MeV

(s-shell ΛN interaction: $\bar{V}_s + \Delta_s s_N.s_\Lambda$)

$\Lambda - \Sigma$ Nijmegen model dependence



$\Lambda - \Sigma$ Coherent Coupling

($1s_\Lambda \to 1s_\Sigma$ & same nucleon orbital wavefunction)

- $\Lambda\Sigma$ coupling: $\sqrt{4/3}\,(\, t_N \cdot t_Y \, \overline{V}' + \, s_N \cdot s_Y \; t_N \cdot t_Y \, \Delta' \,)$ ($\sqrt{4/3}$ arises from t_Y changing Λ to Σ) **leading to Fermi & Gamow-Teller (GT) nuclear matrix elements.**
- **The important $\Lambda\Sigma$ coupling matrix elements involve Σ and Λ hyperons coupled to the same nuclear core, and nuclear states connected by a large GT matrix element to the dominant core state.**
- **Sizable $\Lambda\Sigma$ matrix elements arise in realistic models,**

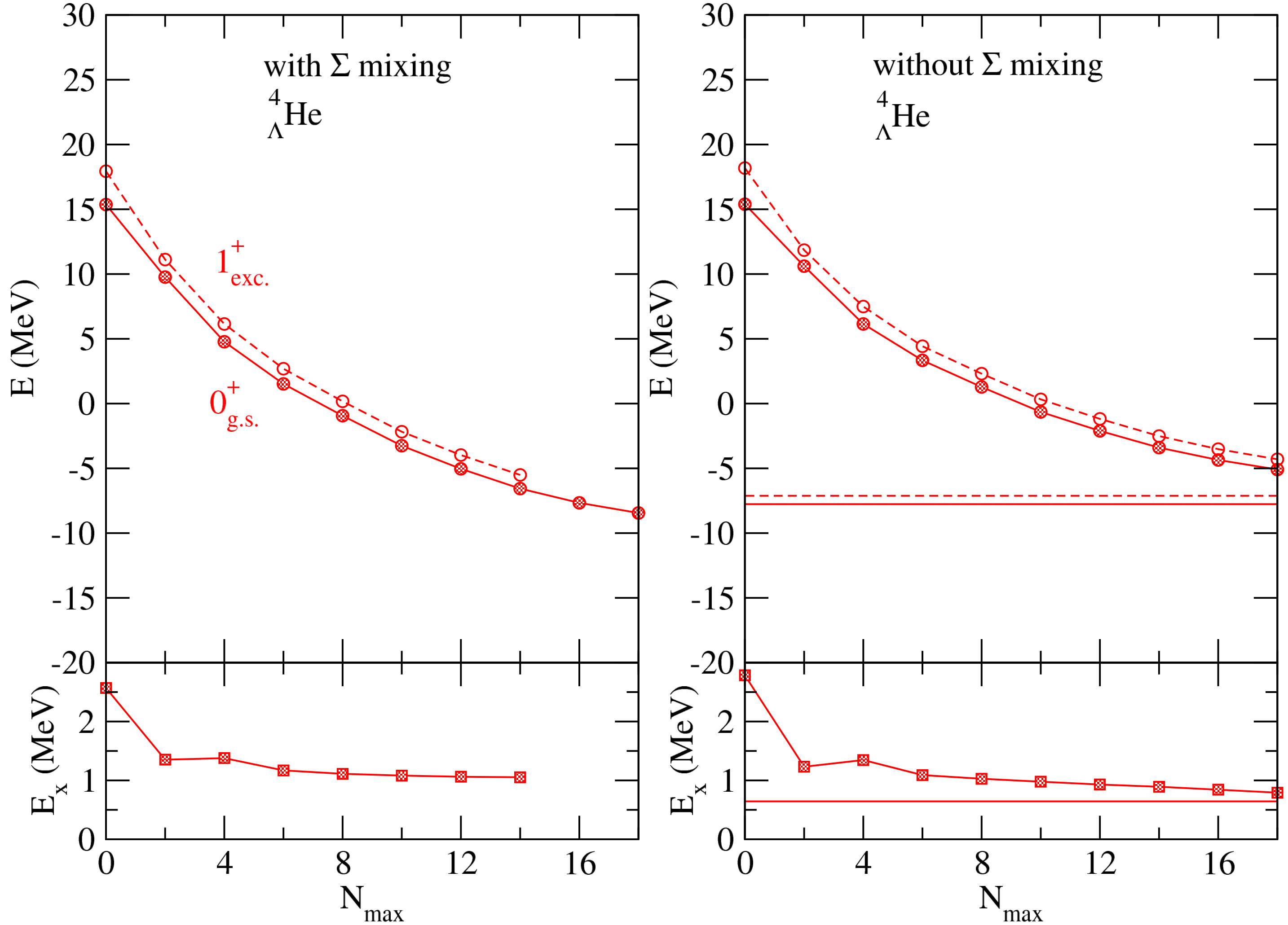
  for example in models NSC97e(f):

$$\bar{V}_{\Lambda\Sigma} = 2.96\ (3.35), \quad \Delta_{\Lambda\Sigma} = 5.09\ (5.76)\ \text{MeV}.$$

$\Lambda\Sigma$ Fermi (F) & Gamow-Teller (GT) matrix elements and binding-energy contributions (MeV)

	${}^{3}_{\Lambda}$H			${}^{4}_{\Lambda}$H	
	$0,\frac{1}{2}^{+}$	$0,\frac{3}{2}^{+}$	$1,\frac{1}{2}^{+}$	$\frac{1}{2},0^{+}$	$\frac{1}{2},1^{+}$
ΛN $(\times\Delta_{\Lambda\Lambda})$	1	$-1/2$	–	$3/4$	$-1/4$
F $(\times\bar{V}_{\Lambda\Sigma})$	–	–	$2\sqrt{2/3}$	1	1
GT $(\times\Delta_{\Lambda\Sigma})$	$\sqrt{3}/2$	–	$-1/2$	$3/4$	$-1/4$
$\frac{1}{80}(\lvert\mathrm{F}\rvert^2+\lvert\mathrm{GT}\rvert^2)$	0.243	–	0.373	–	–
$\frac{1}{80}\lvert(\mathrm{F}+\mathrm{GT})\rvert^2$	–	–	–	0.574	0.036

- In SU(4) limit where nn is as bound as the deuteron $B_{\Lambda}^{T=1}(\frac{1}{2}^{+}) - B_{\Lambda}^{T=0}(\frac{1}{2}^{+}) = (0.373 - 0.243) - \Delta_{\Lambda\Lambda}$.
- Replace $\Delta_{\Lambda\Lambda}$ with increased $\Lambda\Sigma$ contribution by keeping $E(1^{+}) - E(0^{+}) \approx 1.1$ MeV in ${}^{4}_{\Lambda}$H.



LO chiral potentials, private communication Daniel Gazda.

$B(^2\mathrm{n})$ & $B_\Lambda(^3_\Lambda\mathrm{n})$ from Λnn Faddeev equations

$a_s(nn)$ (fm)	$r_s(nn)$ (fm)	$B(^2\mathrm{n})$ (MeV)	$B(^2\mathrm{n})_{\mathrm{approx}}$	$B_\Lambda(^3_\Lambda\mathrm{n})$ (MeV)
5.4	1.75	2.23	2.24	0.39
5.4	2.25	2.79	2.87	0.27
5.4	2.881	4.98	–	0.16
6.0	2.881	2.86	3.20	0.11
9.0	2.881	0.80	0.80	0.01
13.0	2.881	0.32	0.32	0.003
17.612	2.881	0.16	0.16	–
−17.612	2.881	–	–	–

- $B(^2\mathrm{n})_{\mathrm{approx}} = \frac{\hbar^2}{M_n r_s^2}\left(1 - \sqrt{1 - \frac{2r_s}{a_s}}\right)^2$, $B_\Lambda(^3_\Lambda\mathrm{n}) \ll B(^2\mathrm{n})$.

$^3_\Lambda$n dissolves upon breaking SU(4), for fixed $V_{\Lambda N}$, from $B(^2\mathrm{n}){=}B(d)$ progressively to unbound nn.

${}^{3}_{\Lambda}$n: conclusions

- **The ΛN interactions required to bind ${}^{3}_{\Lambda}$n are inconsistent with Λp scattering cross sections at low energies, with ${}^{3}_{\Lambda}\mathrm{H}_{\mathrm{g.s.}}$ binding energy, and with the $0^{+}_{\mathrm{g.s.}}$–1^{+}_{exc} excitation energy of the $A = 4$ Λ hypernuclei.**
- **The consequences of accepting a bound ${}^{3}_{\Lambda}$n for Λ hypernuclear data are sufficiently strong that the use of more refined interactions is unlikely to modify any of the conclusions reached here.**
- **Could ΛNN interactions bind ${}^{3}_{\Lambda}$n? – unlikely.**
- **Could CSB bind ${}^{3}_{\Lambda}$n? – unlikely.**

Other neutron-rich hypernuclei

${}^{6}_{\Lambda}\mathrm{H}$ phenomenology

- Spin flip is forbidden in production at rest:

  $K^{-}_{\mathrm{stop}} \; + \; {}^{6}\mathrm{Li}(L_i = 0, \; S = 1) \; \to \; {}^{6}_{\Lambda}\mathrm{H}(L_f, \; S = 1) \; + \; \pi^{+}$

  Here, $L_f = 0$, so only ${}^{6}_{\Lambda}\mathrm{H}(1^{+}_{\mathrm{exc.}})$ is produced, followed by

  (i) ${}^{6}_{\Lambda}\mathrm{H}(1^{+}_{\mathrm{exc.}}) \; \to \; \gamma \; + \; {}^{6}_{\Lambda}\mathrm{H}(0^{+}_{\mathrm{g.s.}}) \quad (\sim 10^{-13}\ \mathrm{s})$

  (ii) ${}^{6}_{\Lambda}\mathrm{H}(0^{+}_{\mathrm{g.s.}}) \; \to \; \pi^{-} \; + \; {}^{6}\mathrm{He}(0^{+}_{\mathrm{g.s.}}) \quad (\sim 10^{-10}\ \mathrm{s})$

- If so, $B_{\Lambda}({}^{6}_{\Lambda}\mathrm{H}) = (4.5 \pm 1.2)$ MeV; Is $(1^{+}_{\mathrm{exc.}})$ particle stable?

- Shell-model estimate for Λ interaction with p-shell neutrons:

  $B_{\Lambda}({}^{7}_{\Lambda}\mathrm{He})$-$B_{\Lambda}({}^{5}_{\Lambda}\mathrm{He})$=(5.36±0.09)-(3.12±0.02)=(2.24±0.09) MeV

  and additional $\Lambda N \leftrightarrow \Sigma N$: $\Delta V_{\Lambda N \leftrightarrow \Sigma N}$=0.15 MeV.

  Add to $B_{\Lambda}({}^{4}_{\Lambda}\mathrm{H})$=2.04±0.04 MeV $\Rightarrow$ $B^{\mathrm{SM}}_{\Lambda}({}^{6}_{\Lambda}\mathrm{H})$=4.43±0.10 MeV.

  Scale $\langle V_{\Lambda n} \rangle$ by ×0.8 to fit halo n in ${}^{6}_{\Lambda}\mathrm{He}$: $B_{\Lambda}({}^{6}_{\Lambda}\mathrm{H}) \approx$3.93±0.08 MeV

  $\Rightarrow B_{2n}({}^{6}_{\Lambda}\mathrm{H}) \approx$ 0.2±0.3 MeV, **hardly bound w.r.t. ${}^{4}_{\Lambda}\mathrm{H}$+2$n$.**

Beyond-mean-field $\Delta B_{\Lambda}^{\rm g.s.}$ shell-model contributions (in keV) to normal-parity g.s. of neutron-rich hypernuclei

Millener, NPA 881 (2012) 298; Gal & Millener, PLB 725 (2013) 445

target ^{A}Z	n–rich $^{A}_{\Lambda}(Z-2)$	$\Lambda\Sigma$ diag.	$\Lambda\Sigma$ total	$\Delta B_{\Lambda}^{\rm g.s.}$ total
^{9}Be	$^{9}_{\Lambda}$He$(\frac{1}{2}^{+})$	210	253	879
^{10}B	$^{10}_{\Lambda}$Li(1^{-})	202	275	1022
^{12}C	$^{12}_{\Lambda}$Be(0^{-})	184	158	748
^{14}N	$^{14}_{\Lambda}$B(1^{-})	189	255	785

- Production by $^{A}Z(K^{-},\pi^{+})^{A}_{\Lambda}(Z-2)$ or by $^{A}Z(\pi^{-},K^{+})^{A}_{\Lambda}(Z-2)$
- **Modest $\Lambda\Sigma$ coupling effects from neutron excess.**
- **Beyond-mean-field $\Delta B_{\Lambda}^{\rm g.s.}$ is dominated by ΛN spin dependence, mostly by induced $s_N \cdot \ell_N$.**

$B_\Lambda^{\rm g.s.}$ predictions (in MeV) for n-rich hypernuclei

A. Gal & D.J. Millener, PLB 725 (2013) 445

n-rich	normal	normal	normal	n-rich	n-rich
${}^{A}_{\Lambda}Z$	${}^{A}_{\Lambda}Z'$	$B_\Lambda^{\rm g.s.}$	$\Delta B_\Lambda^{\rm g.s.}$	$\Delta B_\Lambda^{\rm g.s.}$	$B_\Lambda^{\rm g.s.}$
${}^{9}_{\Lambda}\mathrm{He}(\frac{1}{2}^{+})$	${}^{9}_{\Lambda}\mathrm{Li}/{}^{9}_{\Lambda}\mathrm{B}$	8.44±0.10	0.952	0.879	8.37±0.10
${}^{10}_{\Lambda}\mathrm{Li}(1^{-})$	${}^{10}_{\Lambda}\mathrm{Be}/{}^{10}_{\Lambda}\mathrm{B}$	8.94±0.11	0.518	1.022	9.44±0.11
${}^{12}_{\Lambda}\mathrm{Be}(0^{-})$	${}^{12}_{\Lambda}\mathrm{B}$	11.37±0.06	0.869	0.748	11.25±0.06
${}^{14}_{\Lambda}\mathrm{B}(1^{-})$	${}^{14}_{\Lambda}\mathrm{C}$	12.17±0.33	0.904	0.785	12.05±0.33

- **Small $B_\Lambda^{\rm g.s.}$ modifications induced by $\Lambda\Sigma$ coupling.**
- **Small $\Lambda\Sigma$ coupling effects persist also in particle-stable neutron-rich Λ hypernuclei.**

$\Lambda\Sigma$ matrix elements & contributions to $B_{\Lambda}^{\rm g.s.}$ (in MeV) across the periodic table

A. Gal & D.J. Millener PLB 725 (2013) 445

$N-Z$	${}^{A}_{\Lambda}Z$	$\bar{V}_{\Lambda\Sigma}$	$\Lambda\Sigma(\bar{V})$	$\Delta_{\Lambda\Sigma}$	$\Lambda\Sigma(\Delta)$	$\Delta B_{\Lambda}^{\rm g.s.}(\Lambda\Sigma)$
4	${}^{9}_{\Lambda}$He	1.194	0.143	4.070	0.104	0.246
8	${}^{49}_{\Lambda}$Ca	0.175	0.010	0.946	0.014	0.024
22	${}^{209}_{\Lambda}$Pb	0.0788	0.052	0.132	0.001	0.053

- $\Lambda\Sigma$ from Halderson, following NSC97f.
- $\bar{V}_{\Lambda\Sigma}$ **&** $\Delta_{\Lambda\Sigma}$ **decrease drastically as overlap between** $0s$ **hyperon and high-**ℓ **excess neutrons becomes poorer with** A ($0f_{7/2}$ **in** ${}^{49}_{\Lambda}$**Ca,** $0h_{9/2}$ **&** $0i_{13/2}$ **in** ${}^{209}_{\Lambda}$**Pb).**
- **Conclusion:** $\Lambda\Sigma$ **contributes less than 100 keV to binding of medium & heavy n-rich hypernuclei.**